Solving direct and inverse boundary value problems for a piecewise homogeneous medium on physically informed radial basis function networks

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Many boundary value problems are considered in the formulation for piecewise homogeneous media: problems of modeling oil fields [1], thermal conductivity [2], modeling groundwater [3]. In such problems, the properties of the medium are constant in some areas, and at the boundaries of the media, conjugation conditions are satisfied, as a rule, the conditions of ideal conjugation: equality of solutions and flows at the interface between the media. Equality of flows means a discontinuity in the derivative of the solution at the interface between the media.

To solve such problems, it is proposed to use networks of radial basis functions (RBFN), trained by the Levenberg Marquardt algorithm adapted for learning RBFN. RBFN implement a meshless solution method, which does not allow the discontinuity of the derivative of the solution to be realized at the interfaces between media. In addition, radial basis functions with an unlimited domain of definition influence the solution in all regions of the medium. In [4], the authors proposed an approach in which problems for areas with different properties are solved on individual RBFN, and the coupling conditions are taken into account in the loss function.

Inverse boundary value problems for piecewise homogeneous media in a meshless formulation have not been solved previously. To solve the inverse problem in a meshless formulation, we assume that the properties of the medium are approximately described by a continuous differentiable function approximated by the RBFN. The solution to the direct problem, in which the properties of the environment are approximated by a network, is found by the second RBFN. Solving the problem comes down to alternately adjusting the parameters of the two specified networks using the loss function, which includes the sum of squared residuals of the solution at trial points inside and on the boundary of the solution area, as well as at points with a known solution. The iterative regularization method (Morozov condition) was used as a regularizer.

The solution of the model problem showed the possibility of approximate determination of the position of the interface between media and a fairly accurate restoration of the values of the medium function in different areas.

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