Numerical solution to an inverse problem on a hyperbolic heat equation

Akindinov George Dmitrievich
МФТИ (Долгопрудный), Россия
e-mail: akindinov.gd@phystech.edu

Матюхин В. В.

Krivorotko Olga
Институт математики им. С.Л. Соболева (Новосибирск), Россия
e-mail: olga.krivorotko@gmail.com

In this paper we describe an algorithm of numerical solving of an inverse problem on a general case of a hyperbolic heat equation with additional second time derivative with a small parameter. The problem in this case is finding an initial distribution with given final distribution. This algorithm allows finding a solution to the problem for any admissible given precision. Algorithm allows evading difficulties analogous to the case of heat equation with inverted time. Furthermore, it allows finding an optimal grid size by learning on a relatively big grid size and small amount of iterations of a gradient method and later extrapolates to the required grid size using Richardson's method. This algorithm allows finding an adequate estimate of Lipschitz constant for the gradient of the target functional. Finally, this algorithm may easily be applied to the problems with similar structure, for example in solving equations for plasma, social processes and various biological problems. The theoretical novelty of the paper consists in the developing of an optimal procedure of finding of the required grid size using Richardson extrapolation in context of ill-posed problems, and accelerated methods were applied to this problem. Furthermore, we investigated how the algorithm works on data with added noise, and we suggested some other ideas on how to improve the algorithm using adaptive step size for gradient method, and the connection of the problem to machine learning.