Optimal control of the motion of a viscous heat-conducting gas using physics-informed neural networks

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OPTIMAL CONTROL OF THE MOVEMENT OF VISCOUS HEAT-CONDUCTING GAS USING NEURAL NETWORKS

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The optimal control problem for a system of gas dynamics equations in the one-dimensional case is considered. The speed of the medium at the initial moment of time and at the right end of the boundary is selected as control. A mathematical model describing the movement of a viscous heat-conducting gas in the interval $(0,1)$, together with the boundary and initial conditions, can be described by the following system of equations:

$$\rho\left[u_t + \mathrm{Sh}\, u u_x\right] = \frac{\mathrm{Sh}}{\mathrm{Re}} u_{xx} - \mathrm{Sh}\, k\, (\rho\theta)_x = 0, \quad \rho_t + \mathrm{Sh}\, (u\rho)_x = 0, \tag{1}$$

$$\rho\left[\theta_t + \mathrm{Sh}\, u\theta_x\right] = \frac{\mathrm{Sh}}{\mathrm{Pe}}\theta_{xx} + \frac{\mathrm{Sh}}{\mathrm{Re}}\frac{\pi}{k}(u_x)^2 - \mathrm{Sh}\,\pi\,\rho\theta u_x = 0, \tag{2}$$

$$u|_{t=0} = u_0(x), \quad \rho|_{t=0} = \rho_0(x), \quad \theta|_{t=0} = \theta_0(x), \tag{3}$$

$$u|_{x=0} = u_1(t), \quad \rho|_{x=0} = \rho_1(t), \quad \theta|_{x=0} = \theta_1(t), \quad u|_{x=1} = u_2(t), \quad \theta|_{x=1} = \theta_2(t), \tag{4}$$

where u, ρ, θ are unknown gas speed, density and temperature, $u_t = \partial u/\partial t$, $u_x = \partial u/\partial x$, $u_{xx} = \partial^2 u/\partial x^2$, u_0, ρ_0, θ_0, u_1, ρ_1, θ_1, u_2, θ_2 — given functions, Sh, Pe, Re, π, k — dimensionless coefficients.

The optimal control problem comes down to minimizing the following quality functional:

$$J(\mathbf{v}) = J_f(\mathbf{s}) + \alpha_1 \int_0^1 |u_{0x}|^2\, dx + \alpha_2 \int_0^1 |u_{2t}|^2\, dt, \tag{5}$$

where $\mathbf{s}=\{u,\rho,\theta\}$ — state of the system (1)–(4), $\mathbf{v}=\{u_0,u_2\}$ — control, $\alpha_1 > 0$, $\alpha_2 > 0$, $J_f(\mathbf{s})$ — lower semicontinuous functional.

The correctness of the optimal control problem (1)–(5) was studied in the work [1].

Based on the neural network approximation of unknown functions, an algorithm has been developed for finding an approximate solution to the extremal problem (5) for various target functionals $J_f(\mathbf{s})$.

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СПИСОК ЛИТЕРАТУРЫ

1. *Amosova E.V* Optimal control of the flow of viscous heat-conducting gas // Siberian Journal of Industrial Mathematics, 2007, volume 10, number 2, pp. 5–22.