# REGULARISATION OF ILL-POSED ALGEBRAIC SYSTEMS

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The solution of ill-posed systems of linear equations has been an active and important area of research.

We discuss:

$$Az = u, \quad z \in Z = R^n, \quad u \in U = R^n. \tag{1}$$

(1)If it is well-conditioned, i.e., the number of conditions is small, the solution of equation (1) is stable with respect to the perturbation of the right terminal term; thus the problem is well-posed.

(2)If it is ill-posed, i.e., when the condition number is very large, a small change in the right-hand term $u$ will cause a large deviation of the approximate solution from the true solution, and thus equation (1) is ill-posed.

**Theorem 1.**The vector $x_T \in U$ is said to be a regular fitting solution to the system of equations (1), which satisfies

$$\|x_T\| = \min_{u \in U} \|u\| .$$

**Theorem 2.**Let $A$ be a continuous operator from the metric space $F$ to the metric space $U$. Then $\forall \alpha > 0$ and $\forall u \in U, \exists z_\alpha \in F_1$ such that the generalized function

$$M^\alpha [z, u] = \rho_u^2 (Az, u) + \alpha \Omega [z], u \in U, z \in F_1 \subset F$$

reaches its lower exact bound at $z_\alpha$, i.e.

$$M^\alpha [z_\alpha, u] = \inf_{z \in F_1} M^\alpha [z, u] .$$

**Theorem 3.**The eigenvalues of $\tilde{A}^T \tilde{A}$ are $\bar{\lambda}_1, \bar{\lambda}_2, \cdots, \bar{\lambda}_n$. For $A^T A$ the size of the eigenvalue perturbation is the same as $\| A - \tilde{A} \|$.

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## References

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