Forward and Inverse Problems of Mean Field Games via Carleman Estimates

Presentation on
XV international scientific conference and young scientist school Theory and Numerics of Inverse and Ill-posed Problems
Akademgorodok, Novosibirsk, October 30 – November 6, 2023

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In this talk, we will present our recent results of [8]-[13].

The mean field games (MFG) theory is a relatively new field, which studies the collective behavior of large populations of rational decision-makers. This theory was first introduced in 2006-2007 in seminal publications of Lasry and Lions [14] as well as of Huang, Caines and Malhamé [3]. Social sciences enjoy a rapidly increasing role in the modern society. Therefore, mathematical modeling of social phenomena can potentially provide a quite important societal impact. In this regard, the MFG theory is the single mathematical model of social processes, which is based on an universal system of coupled Partial Differential Equations (PDEs) [2]. That system is the so-called Mean Field Games system (MFGS). The number of applications of the MFGS to the societal problems is flourishing and includes such areas as, e.g. finance, fight with corruption, cybersecurity, quantum information theory, election dynamics, robotic control, etc.

Thus, due to a broad range of applications of the MFGS, it is important to address various mathematical questions for this system. In the series of six recent publications in 2023 the presenter with co-authors has addressed questions of uniqueness and stability of various forward and inverse problems for the MFGS [8]-[13]. More precisely, Hölder and Lipschitz stability results are proven for these problems. They imply uniqueness. We use the term "forward problem" in the case when the coefficients of the MFGS are known and it is required to determine the solution of the MFGS using some initial, terminal and boundary conditions. We use the term "Coefficient Inverse Problem" if it is required to determine a coefficient of the MFGS, given Dirichlet and Neumann data at a part of the boundary and some initial and terminal conditions.

In fact, we brought in the ideology of theories of Ill-Posed and Inverse Problems in the theories of both forward and inverse problems for the MFGS.

All results of [8]-[13] are obtained using the apparatus of Carleman estimates. Historically, Carleman estimates were first introduced in the field of Coefficient Inverse Problems in the work of Bukhgeim-Klibanov in 1981 [1]. The framework of [1] has been broadly used since then for proofs of global uniqueness and stability results for Coefficient Inverse Problems, see, e.g. [4, 5, 7, 15, 16] and references cited therein for some follow up publications. The presenter with coauthors has also extended the idea of [1] from the theory to globally convergent numerical methods for Coefficient Inverse Problems see, e.g. [6, 7].

References

[1] Bukhgeim, A.L. and Klibanov, M.V.: Uniqueness in the large of a class of multidimensional inverse problems, Soviet Mathematics Doklady, 17, 244–247 (1981).

[2] Burger, M., Caffarelli, L. and Markowich, P.A.:Partial differential equation models in the socio-economic sciences, Philosophical Transactions of Royal Society, A372, 20130406 (2014).

[3] Huang, M., Caines, P.E, and Malhamé, R.P.: Large-population cost-coupled LQG problems with nonuniform agents: individual-mass behavior and decentralized Nash equilibria, IEEE Trans. Automat. Control, 52, 1560–1571 (2007).

[4] Imanuvilov, O.Y. and Yamamoto, M.: Lipschitz stability in inverse parabolic problems by the Carleman estimate, Inverse Problems, 14, 1229-1245 (1998).

[5] Klibanov, M.V.: Carleman estimates for global uniqueness, stability and numerical methods for coefficient inverse problems, J. of Inverse and Ill-Posed Problems, 21, 477-510 (2013).

[6] Klibanov, M.V., Li, J. and Zhang, W.: Convexification for an inverse parabolic problem, Inverse Problems, 36, 085008 (2020).

[7] Klibanov M.V. and Li, J.: Inverse Problems and Carleman Estimates: Global Uniqueness, Global Convergence and Experimental Data, De Gruyter, Berlin (2021).

[8] Klibanov M.V. and Averboukh, Y.: Lipschitz stability estimate and uniqueness in the retrospective analysis for the mean field games system via two Carleman estimates, arXiv: 2302.10709 (2023).

[9] Klibanov, M.V.: The mean field games system: Carleman estimates, Lipschitz stability and uniqueness, Journal of Inverse and Ill-Posed Problems, published online, https://doi.org/10.1515/jiip-2023-0023 (2023).

[10] Klibanov, M.V., Li, J. and Liu, H.: On the mean field games system with lateral Cauchy data via Carleman estimates, *arXiv*: 2303.0758 (2023).

[11] Klibanov, M.V., Li, J. and Liu, H.: Hölder stability and uniqueness for the mean field games system via Carleman estimates, *arXiv*: 2304.00646, 2023.

[12] Klibanov, M.V., Li, J. and Liu, H.: Coefficient inverse problems for a generalized mean field games system with the final overdetermination, arXiv: 2305.01065 (2023).

[13] Klibanov, M.V.: A coefficient inverse problem for the mean field games system, https://arxiv.org/search/?query=Klibanov&searchtype=all&source=header (2023).

[14] Lasry, J.-M. and Lions, P.-L.: Mean field games, Japanese Journal of Mathematics, 2, 229-260 (2007).

[15] Rakesh and Salo, M.: The fixed angle scattering problem and wave equation inverse problems with two measurements, Inverse Problems, 36, 035005 (2020).

[16] Yamamoto, M.: Carleman estimates for parabolic equations. Topical Review, Inverse Problems, 25, 123013 (2009).