

# NUMERICAL STUDY INTO STOCK MARKET CRISES BASED ON MEAN FIELD GAMES APPROACH

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We present an approach to describe the stock market crises based on Mean Field Games and Optimal Control theory with a turnpike effect. The impact of the large amount of high-frequency traders (HFTs) can be modelled via mean field term. A Mean Field Game is a coupled system of PDEs: a Kolmogorov–Fokker–Planck equation, evolving forward in time and describing evolution of the HFTs probability density function spread by the amount of asset shares; and a Hamilton–Jacobi–Bellman equation, evolving backwards in time and defining the strategy of the HFTs. These equations form a boundary value problem. A coupling condition of this system is  $\alpha = \frac{1}{2k_t} \frac{\partial u}{\partial x}$ :

$$\begin{cases} \frac{\partial m}{\partial t} - \frac{\sigma_t^2}{2} \frac{\partial^2 m}{\partial x^2} + \frac{1}{2k_t} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} m \right) = 0, \\ \frac{\partial u}{\partial t} + \frac{\sigma_t^2}{2} \frac{\partial^2 u}{\partial x^2} + \frac{1}{4k_t} \left( \frac{\partial u}{\partial x} \right)^2 - \lambda_t (x - \tilde{a}_t)^2 = -V(m), \\ m(0, x) = m_0(x), \\ u(T, x) = 0, \end{cases}$$

where  $m(t, x)$  is a probability density function of HFTs,  $x_t \in \mathbb{R}$  is the amount of asset shares held by HFT at time  $t \in [0, T]$ ,  $\tilde{a}_t \in \mathbb{R}$  is a turnpike function,  $k_t > 0$ ,  $\lambda_t > 0$ . The functions  $\tilde{a}_t$ ,  $k_t$ ,  $\lambda_t$  rely on the changes of the asset share price  $S_t$ . The asset share price is based on the geometric Brownian motion that is used to model stock prices in the Black–Scholes model in mathematical finance. The ill-posedness of this PDEs system comes from a turnpike effect. To avoid these difficulties we use a variational approach that is based on the reduction of a PDEs system to a special problem of maximizing the functional with restrictions on the Kolmogorov–Fokker–Planck equation. The functional in extremal problem poses in such a form that the Lagrange multipliers for the constraints satisfy the original Hamilton–Jacobi–Bellman equation and the boundary condition at the final horizon time. We apply the Mean Field Games approach to describe the Chinese stock market crash in 2015 considering the representative stock of CITIC Securities (ticker 600030). We consider the HFTs that form dominating bull and bear market. As a result, the bull strategy imitators do not make any profit.

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