ON INVERSION OF THE RAY TRANSFORMS, ACTING ON 2D TENSOR FIELDS, USING SV-DECOMPOSITION

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Let $B = \{x = (x_1, x_2) \in \mathbb{R}^2 : |x| < 1\}$ denote the unit disk and ∂B be its boundary. We further introduce the following notation for the cylinder $Z = \{(s, \alpha) \in \mathbb{R}^2 : s \in (-1, 1), \alpha \in [0, 2\pi)\}$. Unit vectors $\xi = (\cos\alpha, \sin\alpha)$, $\eta = \xi^\perp = (-\sin\alpha, \cos\alpha)$, $\alpha \in [0, 2\pi)$ and $s \in \mathbb{R}$ define a line $L_{\xi,s} = \{x \in \mathbb{R}^2 : x = s\xi + t\eta,\, t \in \mathbb{R}\}$.

The *ray transforms* $\mathcal{P}_m^{(j)} : H^k(S^m(B)) \to H^k(Z)$, $m \geqslant 1$, $0 \leqslant j \leqslant m$, of a symmetric m-tensor field $\mathbf{w}(x) = \big(w_{i_1 \ldots i_m}(x)\big) \in H^k(S^m(B))$ are given by

$$\big[\mathcal{P}_m^{(j)} \mathbf{w}\big](s, \alpha) = \int\limits_{-\sqrt{1-s^2}}^{\sqrt{1-s^2}} \sum_{i_1, \ldots, i_m = 1}^{2} w_{i_1 \ldots i_m}(s\xi + t\eta)\xi_{i_1} \ldots \xi_{i_j} \eta_{i_{j+1}} \ldots \eta_{i_m} dt.$$

For $j = 0$ the transform is the *longitudinal ray transform*, for $j = m$ the operator is the *transverse ray transform* and for $1 \leqslant j \leqslant m - 1$ the ray transform is called *mixed*.

We consider the integral geometry problem of finding a symmetric m-tensor field or its parts provided that the ray transforms are known. In other words, we look for the inverses of longitudinal, transverse, or mixed ray transforms applied to the tensor field. That is, it is required to solve an operator equation of the form $Af = g$, where A is a linear bounded (integral in our case) operator, f is an unknown symmetric m-tensor field and g is a known right side, which is the initial data of the problem.

In to the singular value decomposition method for inverting operators, the operator A is represented as a series of singular values and basis elements in the space of images. Then the (pseudo)inverse operator is represented, using singular functions, as a series of similar structure with preimages of the basis elements and the inverse of the singular values. The decompositions have been constructed for the operators of ray transforms of vector [1, 2] and 2-tensor [3] fields. In the current report the proposed solution of the problem in the m-tensor case is based on the possibility of representing the corresponding fields using potentials, which are constructed as a product of harmonic functions and classical orthogonal polynomials, as in [1, 3].

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