Collapse of the attractive Bose-Einstein condensates

Yaroslav Kharkov, Evgeny Kuznetsov

Novosibirsk State University, Novosibirsk, Russia
P.N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow, Russia
L.D. Landau Institute for Theoretical Physics, Chernogolovka, Russia

June 7, 2012

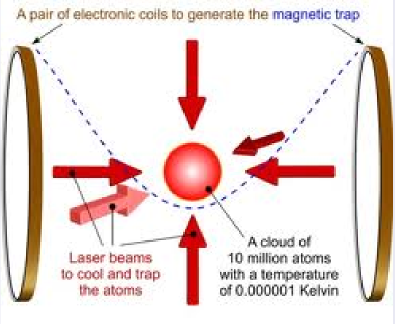
BEC with repulsive interactions

- Predicted by Einstein for non-interacting bosons (1924)
- First experimental observation of BEC in dilute gases of alkali atoms with repulsive interactions (Anderson et al, 1995; Davis et al, 1995)

Experimental setup: cooling and trapping neutral atoms

1. Laser cooling, RF cooling: ($T \sim 1 \mu K$)
2. Magnetic or optical trapping in 3D: parabolic anisotropic potential

$$\boxed{V_{trap}(\rho, z) = \frac{m\omega_{||} z^2}{2} + \frac{m\omega_{\perp} \rho^2}{2}}$$



Particle interactions in BEC

Interaction parameter - scattering length

$$\boxed{a_s: \quad U_0 \equiv \int U_{12}(\vec{r}_{12}) d\vec{r}_{12} = \frac{4\pi\hbar^2 a_s}{m}}.$$

- Repulsive interaction→ positive a_s
- Attractive interaction→ negative a_s

Gross-Pitaevsky equation (GP): $i\hbar\psi = -\frac{\hbar^2}{2m}\Delta\psi + V_{trap}\psi + U_0|\psi|^2\psi$

Trapped condensate is stable for repulsive interatomic interactions!

Attractive BEC

Does any stable BEC for $a_s < 0$ could exist?

Attractive BEC

Does any stable BEC for $a_s < 0$ could exist?

- If $\hbar\omega \gg U_0|\psi|^2$ - system of linear oscillators
- If $\hbar\omega \ll U_0|\psi|^2$ - nonlinear scenario

Attractive BEC

Does any stable BEC for $a_s < 0$ could exist?

- If $\hbar\omega \gg U_0|\psi|^2$ - system of linear oscillators
- If $\hbar\omega \ll U_0|\psi|^2$ - nonlinear scenario

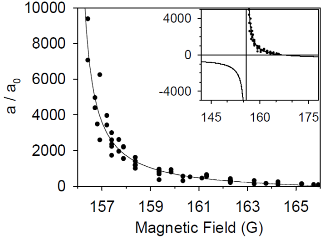
Yes! If the particles number $N < N_{cr}(\omega, U_0)$. $N > N_{cr}(\omega, U_0)$ - collapse!

Experiments with Li^7 $(a_s = -29.2a_b)$:

1. C.A. Sackett et al, "Measurements of Collective Collapse in a Bose-Einstein Condensate with Attractive Interactions", Phys. Rev. Lett. 82, 876 (1999).
2. J M. Gerton et al "Direct observation of growth and collapse of a Bose–Einstein condensate with attractive interactions", Nature 408, 692-695 (2000)

Controlled BEC collapse

Feschbach resonance: $a_s = a_s(B)$



Switching a_s from positive to negative! - Controlled collapse.

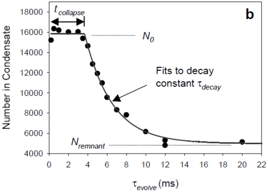
"Bosenova" experiments:

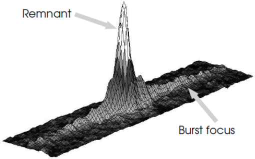
1. (JILA, Colorado, 2001): E. A. Donley et al, "Dynamics of collapsing and exploding Bose–Einstein condensates", Nature 412, 295-299 (2001)
2. (Australia, 2011): P. A. Altin et al, "Bosenova and three-body loss in a Rb-85 Bose-Einstein condensate", arXiv:1108.2561v1

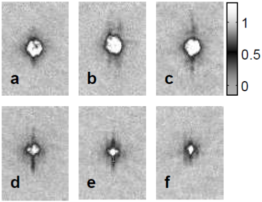
Collapse dynamics

Main features of collapse:

- Missing atoms + burst atoms + jets + remnant condensate







Lack of the reliable theoretical model

Theoretical models

- Quantum many-body problem:
  $i\hbar\partial_t\hat{\Psi} = \left(-\frac{\hbar^2}{2m}\Delta + V_{trap} + U_0\hat{\Psi}^\dagger\hat{\Psi}\right)\hat{\Psi}$
- Mean field approximation ($N_c \gg 1$): $\hat{\Psi} = \psi_0$ → GP equation:
  $i\hbar\partial_t\psi_0 = \left(-\frac{\hbar^2}{2m}\Delta + V_{trap} + U_0|\psi_0|^2\right)\psi$

At $r \sim a_s$ GP breakes down!

Corrections to the GP equation:

- 3 body recombination (dimer + 1 escaping atom)
  $i\hbar\partial_t\psi_0 = \left(-\frac{\hbar^2}{2}\Delta + V_{trap} + U_0|\psi_0|^2 + iK_3|\psi_0|^4\right)\psi_0$
- Beyond mean-field: fluctuations $\hat{\Psi} = \psi_0 + \hat{\chi}$

$$\boxed{i\hbar\partial_t\hat{\chi} = \left(-\frac{\hbar^2}{2m}\Delta + V_{trap}\right)\hat{\chi} + U_0\left(2|\psi_0|^2\hat{\chi} + \psi_0^2\hat{\chi}^\dagger\right)}$$

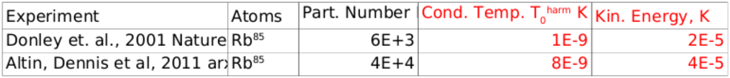
Condensate/uncondensate particles density $n_{cond} = |\psi_0|^2$,
$n_{unc}(\vec{r}) =< \chi^\dagger(\vec{r})\chi(\vec{r}) >$

Energy estimates

What effect is more important? 3-body loss $\propto n_{cond} a_s^3 \ll 1$ gaseous parameter.

Our idea: the uncondensate particles generation is crucial for $r \gg a_s$!

The particles energies are too high! Kinetic energy: $T_{kin} \sim \frac{\hbar^2}{m|a_s|}$ Critical temperature in trap: $T_0^{trap} = \hbar\bar{\omega}N^{\frac{1}{3}}$, $\quad \bar{\omega} = \sqrt[3]{\omega_{||}\omega_{\perp}^2}$



Experiment	Atoms	Part. Number	Cond. Temp. T_0^{harm} K	Kin. Energy, K
Donley et. al., 2001 Nature	Rb^{85}	6E+3	1E-9	2E-5
Altin, Dennis et al, 2011 ar	Rb^{85}	4E+4	8E-9	4E-5

Uncondensate particles generation $\Leftrightarrow$ Collapse stability

Numerical experiment is nedeed. (S. Wuster, J. J. Hope, and C. M. Savage, PRA 2005)

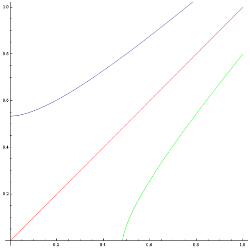
Elliptic deformations instability

Strong quasiclassical collapse in NLSE (Zakharov, Kuznetsov; JETP 1986)

- Generalization - elliptic collapse

$$|\psi_0|^2(\rho, z, t) = \lambda^2 \left[1 - \left(\frac{\rho}{a(t)}\right)^2 - \left(\frac{z}{b(t)}\right)^2\right].$$

- Scaling parameters: effective potential $V_{eff}(a, b) = -\frac{\lambda^2}{a^2 b}$.
- The collapse with $a(t) = b(t)$ is unstable!



Possible explanation for jets(?)

Summary

- The problem of the theoretical description of the attractive BEC collapse is not solved after decade.
- The decoherence effects are underestimated in the existing theoretical approaches.
- Elliptic type instability in strong NLSE collapse is found. Jets (?).