Representation of the Approximate Solutions of the Complete System of Navier-Stokes Equations in One-Dimensional Case

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To describe the motion of a viscous environment is used the traditional system of Navier-Stokes equations in the case of constant density of the environment const ρ=const [1], leads to the fact that for the builded solutions of this system are not fullfill to the law of conservation of energy (in the absence of a system of equations for temperature), or to the basics of thermodynamic identity (for inclusion in the system of equations for temperature).

To describe the flow of a compressible viscous heat-conducting ideal gas in the case of constant coefficients of viscosity and thermal conductivity in this paper we examine a complete system of Navier-Stokes equations (CSNSE) [2]. At the same time as the independent thermodynamic variables are taken specific volume $\delta = 1/\rho$ and pressure *p* [3-5]. In this case CSNSE written in normal form with respect to time derivatives of the unknown functions, and the right sides have a polynomial form with quadratic nonlinearities. This makes it possible to construct approximate solutions CSNSE using ideas of the Galerkin method [6].

For simplicity, we further consider the case of one-dimensional flows. Gasdynamic parameters are presented at such forms:

$$\delta = 1 + \sum_{k=1}^{K} \delta_k(t)\cos(kx), \qquad u = \sum_{k=1}^{K} u_k(t)\sin(kx), \qquad p = 1 + \sum_{k=0}^{K} p_k(t)\cos(kx),$$

where *u* is velocity of the gas. In this case there are the conditions of adhesion and thermal insulation. Substituting representation and the subsequent procedure CSNSE projection of each of the three equations PSUNS on an appropriate system of basis functions leads to recorded in the normal form of a closed system of ordinary differential equations (SODE) for the unknown coefficients $\delta_k(t),\ u_k(t),\ p_k(t)$, (3K+1).

We numerically construct solutions of SODE at different values of the input data. And thus defines the various one-dimensional flow of a compressible viscous heat-conducting gas. In particular, describe the motion of a some traveling shock transitions, which overtake one another.

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