Kinetics of a Network of Vortex Loops in He II and a Theory of Superfluid Turbulence

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A theory is developed to describe the superfluid turbulence on the base of kinetics of the merging and splitting vortex loops. Because of very frequent reconnections the vortex loops as a wholedo not live long enough to perform any essential evolution due to the deterministic motion. On the contrary, they rapidly merge and split, and these random recombination processes prevail over other slower dynamic processes. To develop quantitative description we take the vortex loops to have a Brownian structure with the only degree of freedom, which is the length l of the loop. We perform investigation on the base of the Boltzmann type “kinetic equation” for the distribution function $n(l)$ of number of loops with length l. This equation describes a slow change of the density of loops $n(l)$ in space of their lengths l due to the deterministic equation of motion and due to fast random change because of the frequent reconnections. By use of the special ansatz in the “collision” integral, we have found the exact power-like solution $n^{-5/2}$ of “kinetic equation” in the stationary case. This solution is not (thermodynamically) equilibrium, but on the contrary, it describes the state with two mutual fluxes of the length (or energy) in space of sizes of the vortex loops. The term “flux” means just redistribution of length (or energy) among the loops of different sizes due to reconnections. Analyzing this solution we drew several results on the structure and dynamics of the vortex tangle in the turbulent superfluid helium. In particular, we obtained that the mean radius of the curvature is of the order of interline space. We also evaluated the full rate of the reconnection events. Assuming, further, that the processes of random collisions are the fastest ones, we studied the evolution of full length of vortex loops per unit volume—the so-called vortex line density $L(t)$. It is shown this evolution to obey the famous Vinen equation. The properties of the Vinen equation from the point of view of the developed approach had been discussed. Thus, depending on the temperature (and independently on velocity) vortices either develop into the highly chaotic turbulent state (low temperature), or degenerate into few smooth lines (higher temperature).