What We Know about Integrability of Euler Equations for Deep Fluid with Free Surface?

V. Zakharov

P.N. Lebedev Physical Institute RAS, Moscow, Russia;

Novosibirsk State University, Novosibirsk, Russia

e-mail address: zakharov@math.arizona.edu

The Euler equations describing the two-dimensional non-stationary potential flow of ideal incompressible deep fluid in gravitational field is a teasing and enigmatic object. Since 1994 the arguments in favor of complete integrability of these equations are accumulated. Some of them are analytic. The element of the scattering matrix describing inelastic four-wave interactions is identically zero. Two of nine elements of the scattering matrix describing the five-wave processes are zero as well. The system of equations might have or have not any extra motion constants. The number of these constants depends on initial data. The case of absence of gravity is especially interesting. In this case the evolution of short-scale perturbation on large scale motion is described by well known integrable Laplace Growth Equation (LGE). In the presence of capillarity the stationary propagating waves on deep water are presented by elementary functions. All these facts are indirect evidence in favor of the "deep water integrability" hypothesis. This hypothesis is not proved yet.

In the last years very efficient codes for numerical solution of the free-surface Euler equations have been developed. The numerical experiments also give arguments in support of the integrability conjecture. They show existence of intensive envelope solitons (breathers) with indefinitely long life-time. Moreover, recent experiments show that these solitons collide completely elastically.

The question of homoclinic-type solution of the exact Euler equation is the subject of active consideration now.

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