Formation of Singularities on the Interface of Dielectric Liquids in a Strong Vertical Electric Field

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It is known that a sufficiently strong external electric field directed along the normal to the interface between two dielectric fluids gives rise to instability of the boundary [1]. The exponential growth of the perturbation amplitude of the interface at the initial (linear) stage of the instability development inevitably leads to the situation where nonlinearities play an essential role.

In most publications, nonlinear processes at the interface of dielectric fluids are considered under the assumption that the wavelength is much greater than the depth of the fluid layers (the so-called shallow water approximation), or under the requirement of spectral narrowness of the wave packet, which allows to apply the method of envelopes. In the present work, we demonstrate that the evolution of the interface can be effectively studied analytically without these restrictions in the strong-field limit, where the motion of the interface is determined entirely by the electrostatic forces (the capillary and gravitational forces are neglected).

So, we consider the nonlinear dynamics of the interface of two infinitely deep inviscid incompressible perfect dielectric liquids in a vertical electric field. In the limit of a strong electric field, the integro-differential equations describing the motion of the boundary are derived using the Hamiltonian formalism. These equations take into account linear and quadratic nonlinear terms, so that they describe weakly nonlinear stages of the instability. As it turns out, the equations of motion can be integrated and, consequently, the development of instability can be described analytically in two particular cases: when the ratio of densities of liquids is (i) directly or (ii) inversely proportional to the ratio of their dielectric constants.

In the first case, the root singularities are formed at the fluid interface in a finite time. It is important that, for these weak singularities, the curvature becomes infinite, while the surface itself remains smooth. Such a behavior is similar to the behavior of the charged free surface of a conducting fluid [2]. From a mathematical point of view, the equations describing the instability are similar to those arising in the consideration of the inertial motion of an ideal fluid with free surface [3].

In the second case, there exists the tendency to the emergence and unlimited sharpening of dimples at the interface, that is strong singularities appear for which the slope angle tends to $\pi/2$. It is generally similar to the behavior of the charged boundary of liquid helium (see Ref. [4]).

For arbitrary density and permittivity ratios, the derived equations of motion can not be solved analytically. Nevertheless, the two considered integrable particular cases give a fairly complete picture of the behavior of the interface under the action of the electrostatic forces.

Note that these results can be applied to describe the behavior of the interface between two magnetic fluids in vertical magnetic field.

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