Conservative System of Equations for Spatial Nonlinear Perturbations Modeling on the Surface of Liquid Film Flowing Down the Vertical Wall

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In paper [1] a new system of equations was proposed for modeling the evolution of plane nonlinear perturbations on the free surface of thin film of viscous liquid flowing down a vertical plane. A special coordinate transformation that transfers the flow domain into a strip of constant thickness was used for its derivation from the equations of fluid motion written in tensor notices. The aim of the present study was to develop this approach to the case of spatial perturbations of the liquid film free surface. As the result a new system of equations with corresponding boundary conditions was derived:

$$\frac{\partial(hu)}{\partial t}+\frac{\partial(hu^2)}{\partial x}+\frac{\partial(huv)}{\partial \eta}+\frac{\partial(huw)}{\partial z}=\frac{\sigma}{\rho}\left(\frac{\partial}{\partial x}\left(hh_{xx}-\frac{h_x^2+hh_{zz}}{2}\right)+\frac{\partial}{\partial z}\left(\frac{hh_{xz}-h_xh_z}{2}\right)\right)+\frac{\mu}{h}\frac{\partial^2 u}{\partial \eta^2}+\rho gh$$

$$\frac{\partial(hw)}{\partial t}+\frac{\partial(huw)}{\partial x}+\frac{\partial(hwv)}{\partial \eta}+\frac{\partial(hw^2)}{\partial z}=\frac{\sigma}{\rho}\left(\frac{\partial}{\partial z}\left(hh_{zz}-\frac{h_z^2+hh_{xx}}{2}\right)+\frac{\partial}{\partial x}\left(\frac{hh_{xz}-h_xh_z}{2}\right)\right)+\frac{\mu}{h}\frac{\partial^2 w}{\partial \eta^2}$$

$$\frac{\partial h}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial \eta}+\frac{\partial(hw)}{\partial z}=0$$

$$u=v=w=0,\quad \eta=0$$

$$\frac{\partial u}{\partial \eta}=\frac{\partial w}{\partial \eta}=0,\quad v=0,\quad \eta=1$$

Here *h* is the film height, *u, v, w* are the contravariant components of the fluid velocity vector (longitudinal, transversal, and normal correspondingly), σ, ρ, μ are the surface tension, density and dynamical viscosity of the liquid, respectively.

It was demonstrated that for moderate Reynolds numbers the assumption of self-similarity of the velocity profile allow the system to be integrated across the layer that leads to the well known model [2]. In the case of small Reynolds numbers the solution can be expanded in the series by small parameter of the film height to the characteristic wavelength ratio. Restricting the series one can obtain the well known evolution equation [3] for the deviation of the film height from its equilibrium value.

This work was supported by the Russian Foundation for Basic Research (Grant 10-08-91333) and by the Grant of the Government of Russian Federation designed to support scientific research projects implemented under the supervision of leading scientists at Russian institutions of higher education (No.11.G34.31.0035).

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