On Nonlinear Stage of the Modulational Instability

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In recent time the simplest and most universal model for description of freak (or rogue) waves in ocean and optic fibers is the focusing nonlinear Schrödinger equation (NLSE). There is a point of common agreement that these extreme waves appear as a result of modulational instability of quasimonochromatic nonlinear waves. In terms of the NLSE model we should study instability of the "condensate" in the focusing version of this equation. It is known since 1971 that the NLSE is a system completely integrable by the Inverse Scattering Method. From that time, hundreds of papers and several books have been devoted to this subject. However, the important question, what is nonlinear stage of the modulational instability, is not answered yet.

In this work we study evolution of a special class of localized initial data presented by exact n-solitonic solutions of NLSE. Solitonic solutions in presence of unstable condensate have a long history. The most important publications, which should be mentioned, are [1,2,3].

In this work we started from description of n-solitonic solutions in presence of condensate. We found this solution in exact close analytic form by the use of deressing method. Then we described in details one- and two-solitonic solutions (typical examples of such solutions are presented on **Fig. 1**). Finally we addressed the following question: what kind of solitonic solution can be used as an initial data fore development of modulational instability? Apparently, this is a solution which tends to the same condensate with the same phase at $x \to \pm\infty$. We found sulutions satisfying such criteria.

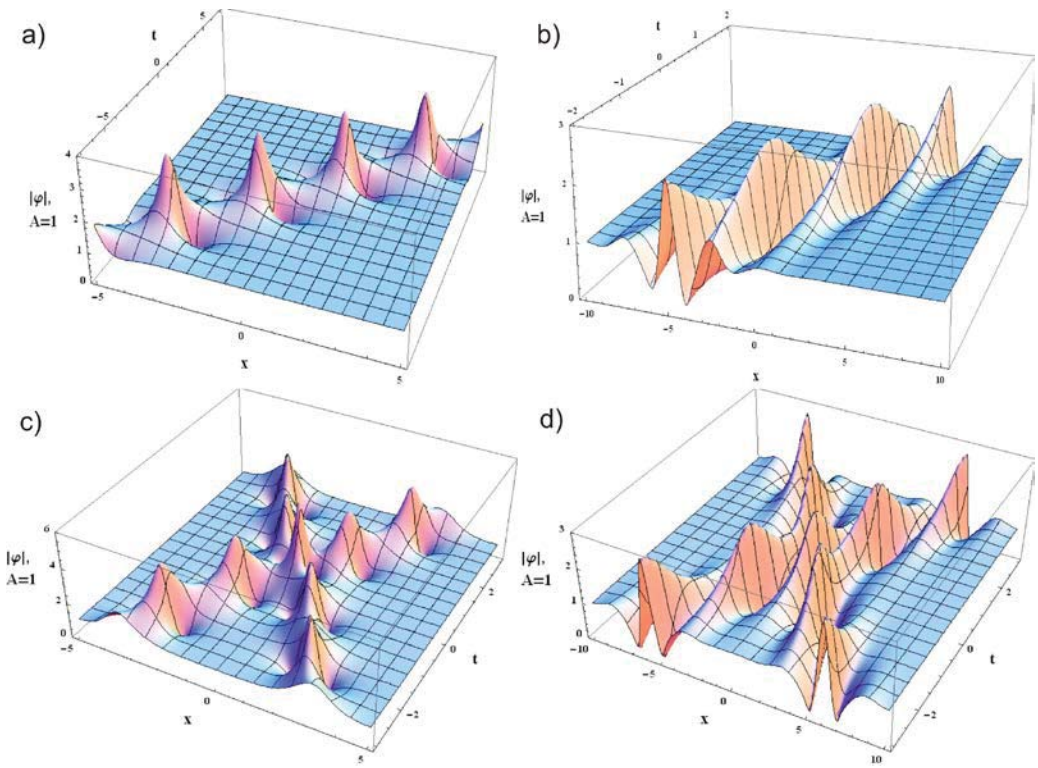

Fig. 1. Typical one - (a,b) and twosolitonic (c,d) solutions with parameters: a) R=2, $\alpha=\pi/12$ b) R=2, $\alpha=5\pi/12$ c) R=3, $\alpha=\pi/12$ d) R=3, $\alpha=\pi/4$.

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