Backward Waves in Negative Index Materials

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Experimental demonstration of the phenomenon of negative index of refraction first in the microwave [1] and latter in the optical regime[2, 3] has stimulated growing interest in nonlinear properties of negative index materials[4]. This interest is motivated by specifics on the interaction of electromagnetic waves with negative index materials. One of the most fundamental properties of negative index material is an opposite directionality of the Poynting vector, characterizing the energy flux, to the wave vector ***k***. On the other hand, the negative index property can be realized only on particular wavelength intervals. These two features are offering a very unusual type of multi-wave interactions, if frequencies of interacting waves correspond to frequency intervals where optical material has different signs of refractive index. Multi-wave interaction must satisfy a phase matching condition, which is possible only when all wave vectors are pointed in the same direction [5]. Therefore energy fluxes of the waves with frequencies corresponding to a negative sign of refractive index will propagate in opposite direction to those with frequencies corresponding to a positive sign of refraction index [6, 7].

We considered three wave interaction in negative index materials. In particular second harmonic generation and amplification, and parametric amplification are studied. We demonstrated that in contrast to the conventional case there is a critical value of phase mismatch in second harmonic generation. If the absolute value of phase mismatch is below critical, then the field intensities are monotonically decaying along the sample, providing efficient conversion of pump wave to a second harmonic. When the absolute value of phase mismatch exceeds a critical value, monotonic decay of intensities transforms to a spatial periodic oscillations. Note, that in the conventional case the critical value of phase mismatch is zero. We showed that presence of critical mismatch leads to a different regimes of second harmonic amplification and dramatically changes form of three wave interaction.

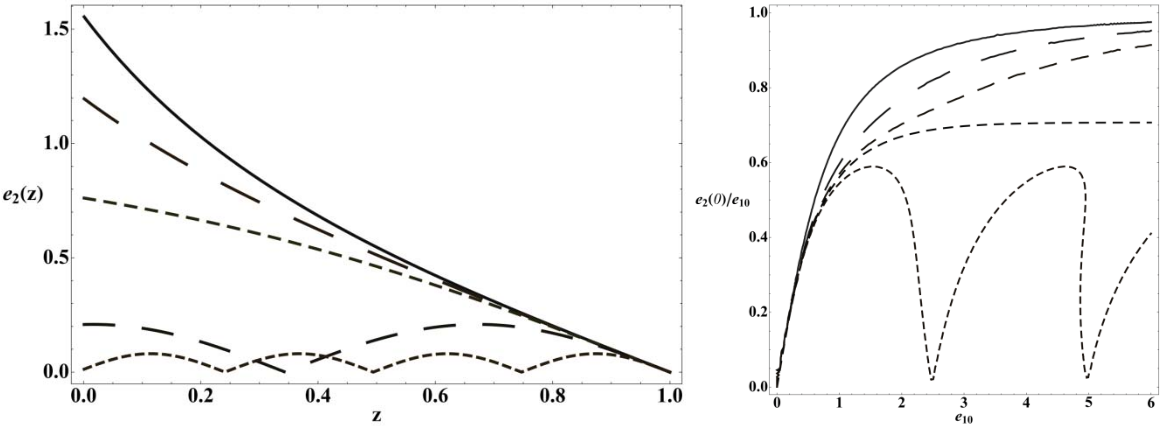

Fig.1. Left insert - the dependence of second harmonic's amplitude $e_2(z)$ on coordinate z with different values of phase mismatch Δ (the solid curve: $\Delta = 0$, large dashed curve $\Delta < \Delta_{cr}$, small dashed curve: $\Delta = \Delta_{cr}$, intermediate dashed curve: $\Delta = 2.5\Delta_{cr}$, doted curve: $\Delta = 6\Delta_{cr}$). Right insert – the dependence of conversion efficiency $\alpha = e_2(0)/e_{10}$ on input field amplitude e_{10} with different values of phase mismatch Δ: Solid curve $\Delta = 0$; large dashed curve $\Delta = 0.88\Delta_{cr}$; dashed curve $\Delta = 0.98\Delta_{cr}$; small dashed curve $\Delta = \Delta_{cr}$; doted oscillation curve $\Delta = 1.13\Delta_{cr}$

The difference in spatial second harmonic amplitude distribution along the sample at different values of phase mismatch Δ is shown in the left insert of the figure above (Fig.1). Left insert shows efficiency of frequency conversion as function of input pump amplitude at different values of phase mismatch.

References

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