Finite Element Modeling of the Surface Water Waves

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Computer simulation of free surface water waves has been implemented by means of finite difference, finite element, boundary integral equation and spectral approaches by many authors, see [1]-[4] for example. This paper concerns the finite element modelling for the Hamiltonian statement which was proposed in the pioneer article [5].

The original PDEs describe 2-D potential flow of inviscid fluid with free surface in computational domain of finite depth, under kinematic and Bernoulli conditions on the upper boundary and periodic conditions on vertical boundaries.

Numerical solution is based on the implicit integration procedure for Hamiltonian differential system with the discrete approximation of the functional derivatives which conserves the total energy at each time step. The inversion of Steklov–Poincare (Dirichlet–Neumann) operator is implemented by finite element solution of the mixed boundary value problem for Laplace equation on the dynamic grids adapted to the variable free surface.

The systems of linear algebraic equations are solved by the parallel domain decomposition method using preconditioned iterative process in Krylov subspaces [6]. Scalable implementation of algorithms is done using hybrid programming on the computer architectures with distributed and shared memory.

The numerical results on wave propagation for several test problems are presented. The performance of code and obtained accuracy are demonstrated on the set of experiments with high resolution grids.

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