Rogue Waves Statistics in the Framework of One-Dimensional Generalized Nonlinear Schrodinger Equation

V.E. Zakharov*[1], *D.S. Agafontsev*[2,3]
[1]P.N. Lebedev Physical Institute RAS, Moscow, Russia
[2]P.P. Shirshov Institute of Oceanology RAS, Moscow, Russia
[3]Novosibirsk State University, Novosibirsk, Russia
**e-mail address: dmitrij@itp.ac.ru*

We examine two nonlinear equations: (1) the focusing generalized Nonlinear Schrodinger equation that takes into account the collapsing six-wave interactions term and also eight-wave interactions term,

$$i\psi_t + \psi_{xx} + |\psi|^2\psi + \alpha|\psi|^4\psi - \beta|\psi|^6\psi = 0, \quad \alpha \ll 1, \quad \beta \ll \alpha; \qquad (1)$$

and (2) the focusing generalized Nonlinear Schrodinger equation accounting for six-wave interactions term as well as dumping terms (linear dissipation and three-photon absorption) and a pumping term,

$$i\psi_t + (1-ia)\psi_{xx} + |\psi|^2\psi + (\alpha+ib)|\psi|^4\psi = ic\psi, \quad \alpha, a, b, c \ll 1, \quad b \ll \alpha; \qquad (2)$$

We solve these equations numerically in the box with periodically boundary conditions. We start from the initial data $\mathbf{\Psi_{t=0} = 1 + \varepsilon(x)}$, where $\mathbf{\varepsilon(x)}$ is a stochastic noise. The development of modulation instability in (1) and (2) leads to formation of one-dimensional wave turbulence. In the integrable case $\boldsymbol{\alpha = \beta = 0}$ or $\boldsymbol{\alpha = a = b = c = 0}$ the turbulence is called integrable and relaxes to one of infinite possible stationary states. Addition of six-wave interactions term leads to appearance of collapses that eventually are regularized by (1) the eight-wave interactions term or (2) the dumping terms. The energy lost during regularization of collapses in (2) is restored by the pumping term. In the latter case the system does not demonstrate relaxation-like behavior.

We measured evolution of spatial spectra $\mathbf{I_k = \langle|\Psi_k|^2\rangle}$ and the PDF of $|\mathbf{\Psi}|^2$, concentrating special attention on formation of "fat tails" on the PDFs. Integrable turbulence appearing as a result of modulation instability of the stationary monochromatic wave leads to an almost Gaussian PDF. Addition of higher-order terms to the classical NLS equation leads to deviations from Gaussian-like PDF when higher waves appear more frequently. These deviations form power-law regions between small and high waves for Eq. (1) (see Fig. 1a) and exponential on $|\mathbf{\Psi}|$ tails for Eq. (2) (see Fig. 1b) and can be detected even when six-wave interactions term is small compared to terms of the classical NLS equation. For Eq. (2) non-Gaussian addition to PDF increases with **α** and disappears at zeroth six-wave interactions coefficient **α=0**.

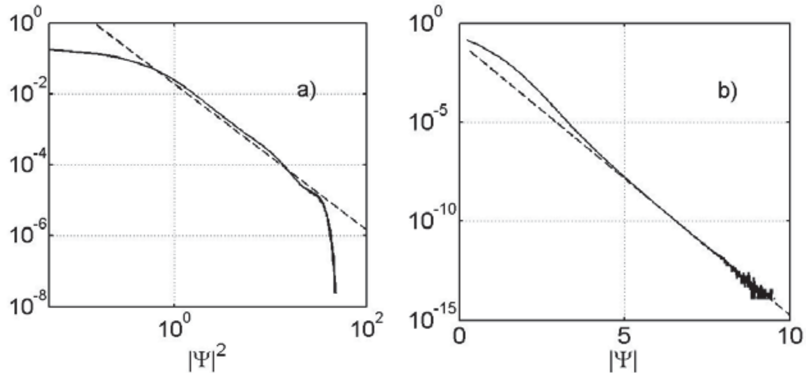

Fig. 1. (a) Power-law region between small and high waves on the PDF of Eq. (1) and (b) ~exp(-c*|**Ψ**|) tail on the PDF of Eq. (2).

This work was fulfilled under foundation by Government of Russian Federation (Grant No. 11.G34.31.0035).

References

[1] D.S. Agafontsev, V.E. Zakharov, “Rogue waves statistics in the framework of one-dimensional Generalized Nonlinear Schrodinger Equation”, arXiv:1202.5763v1.