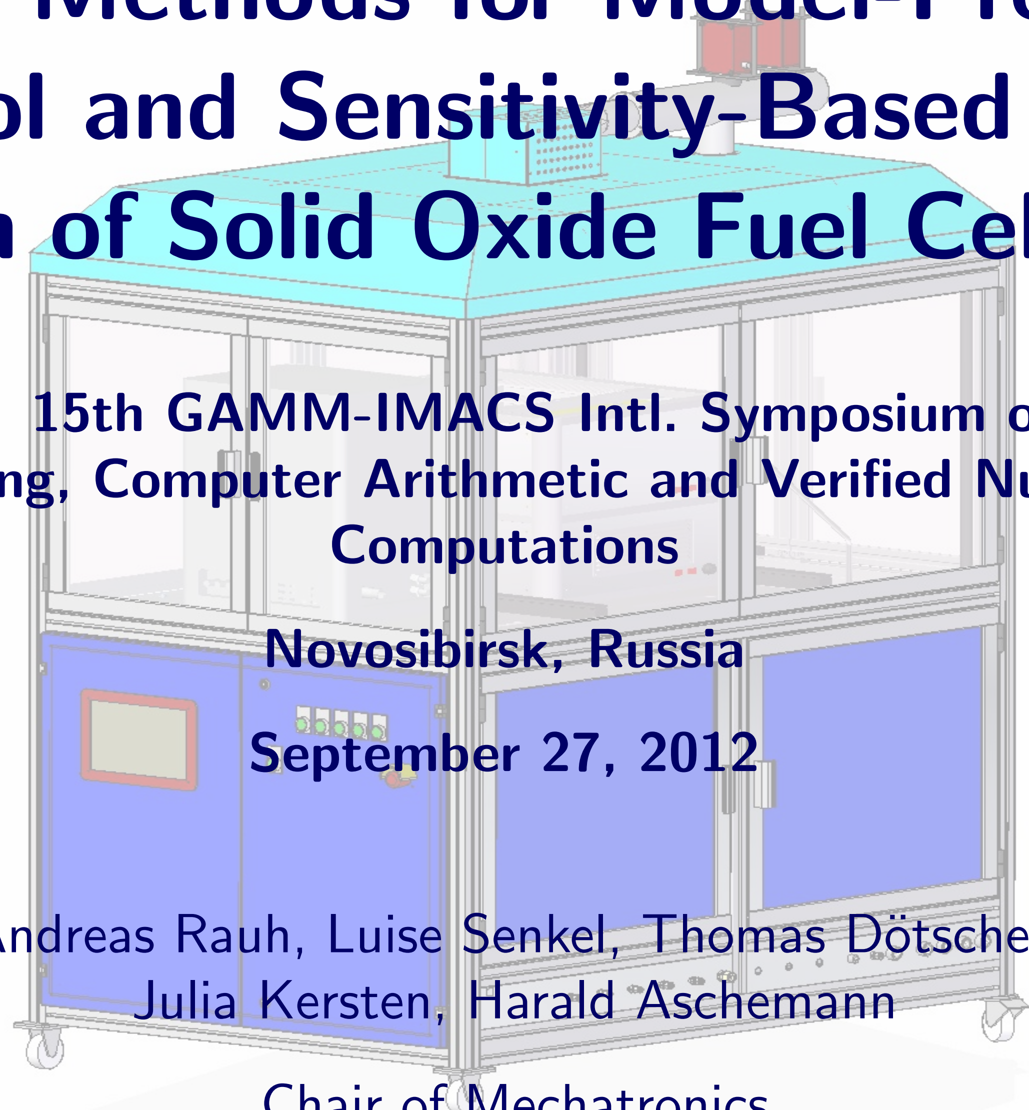
Universität Rostock

Traditio et Innovatio

Chair of Mechatronics

Interval Methods for Model-Predictive Control and Sensitivity-Based State Estimation of Solid Oxide Fuel Cell Systems



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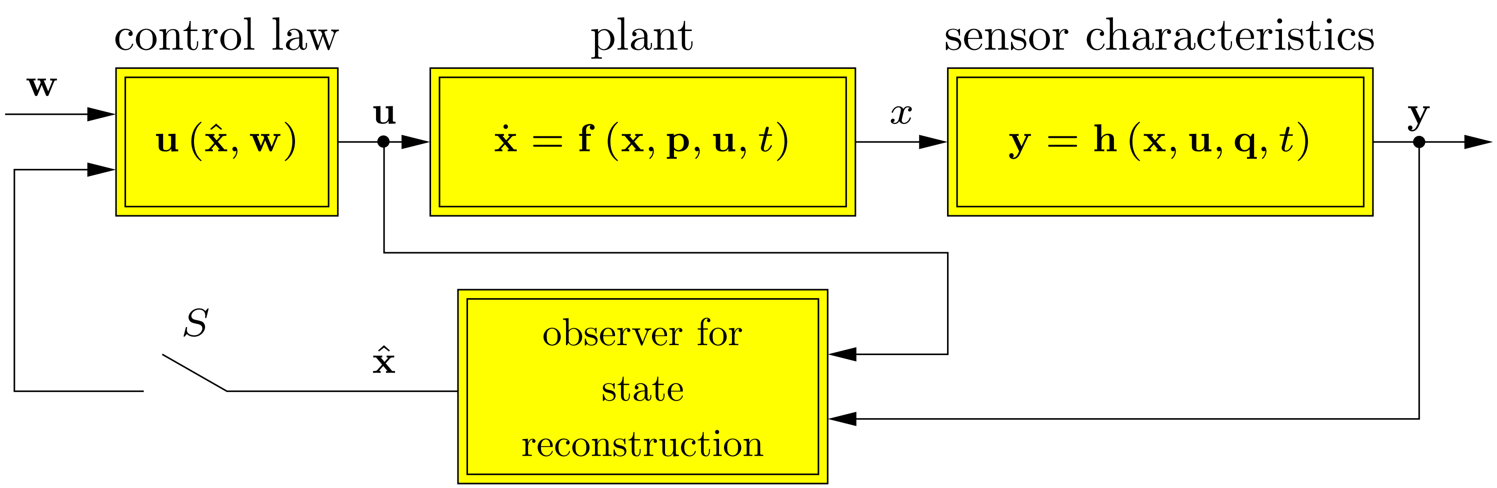
Contents

Tracking Control for Continuous-Time Dynamical Systems

Consider a dynamical system with

- the state equations $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t), t)$
- the output $\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))$, for example, measured data $\mathbf{h}(\cdot)$
- the desired output trajectory $\mathbf{y}_d(t)$



Tracking Control for Continuous-Time Dynamical Systems

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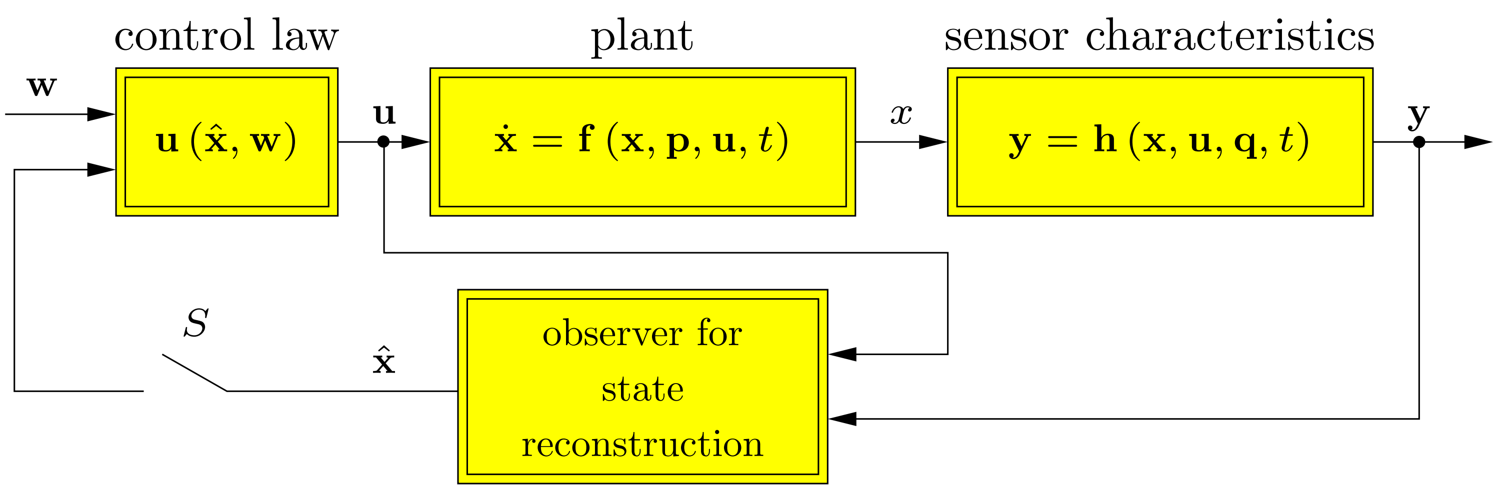

Necessity for state/ output feedback to prevent the violation of feasibility constraints in the case of parameter uncertainties as well as measurement and state reconstruction errors.

Tracking Control for Differentially Flat Systems

Differential Flatness of *Nonlinear* Dynamical Systems $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$

A dynamical system is called differentially flat, if flat outputs

$$\mathbf{y} = \mathbf{y}\left(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \ldots, \mathbf{u}^{(\alpha)}\right)$$

exist such that

(i) all system states $\mathbf{x}$ and all inputs $\mathbf{u}$ can be expressed as functions of the flat outputs and their time derivatives:

$$\mathbf{x} = \mathbf{x}\left(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(\beta)}\right) \quad \text{and} \quad \mathbf{u} = \mathbf{u}\left(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(\beta+1)}\right)$$

(ii) the flat outputs $\mathbf{y}$ are differentially independent, i.e., they are not coupled by differential equations.

Note:

(a) If (i) is fulfilled, (ii) is equivalent to $\dim(\mathbf{u}) = \dim(\mathbf{y})$.

(b) The flat outputs $\mathbf{y}$ need not be the physical outputs of the dynamical system.

(c) For linear systems, differential flatness is equivalent to controllability.

Generalized Tracking Control for Dynamic Systems

- Guaranteed stabilization of the error dynamics by interval evaluation of suitable Lyapunov functions to account for uncertainties
- Transformation of the state equations into nonlinear controller normal form: overcompensation of uncertainties
- Sliding mode control procedures, e.g. evaluated by means of interval analysis: see previous presentation
- Alternatively: Exploitation of inherent robustness properties of model-predictive control procedures

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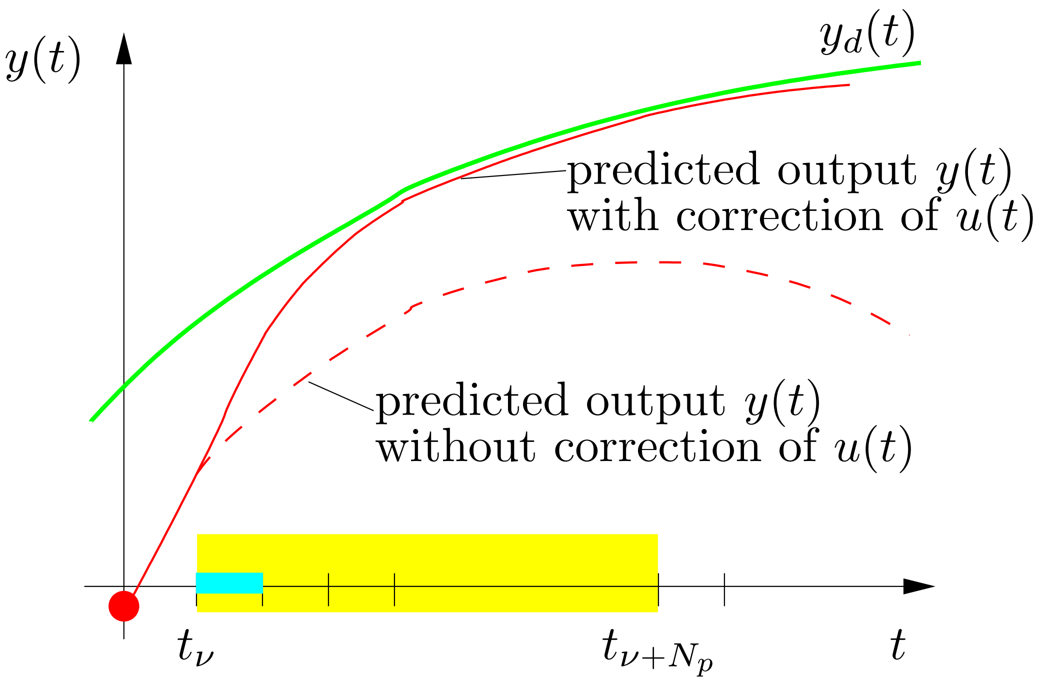
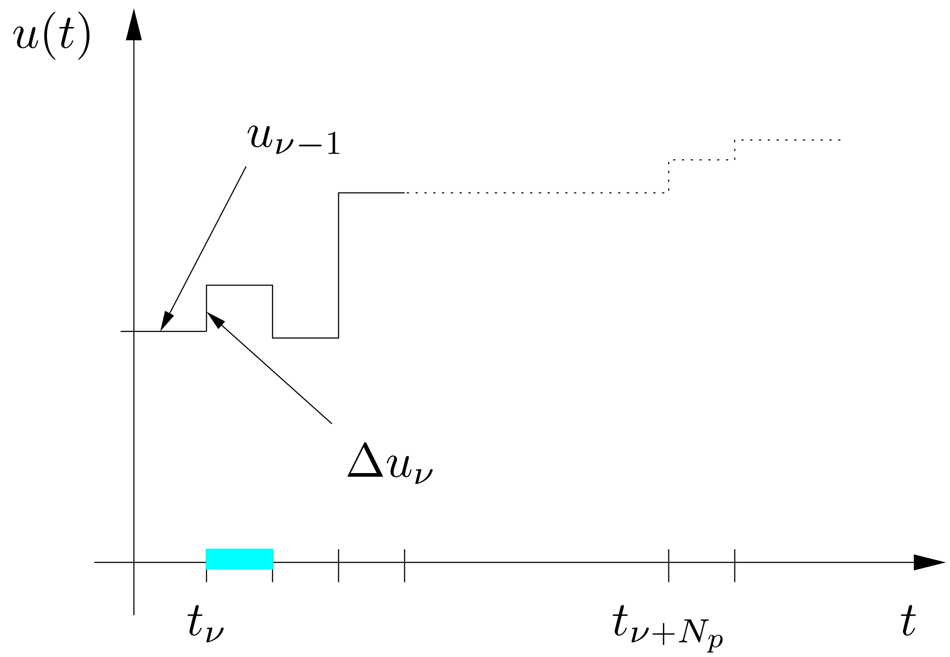
(Interval-based) Predictive control approaches do not require an analytic reformulation of the state equations into a nonlinear controller normal form or into an input-affine system representation.

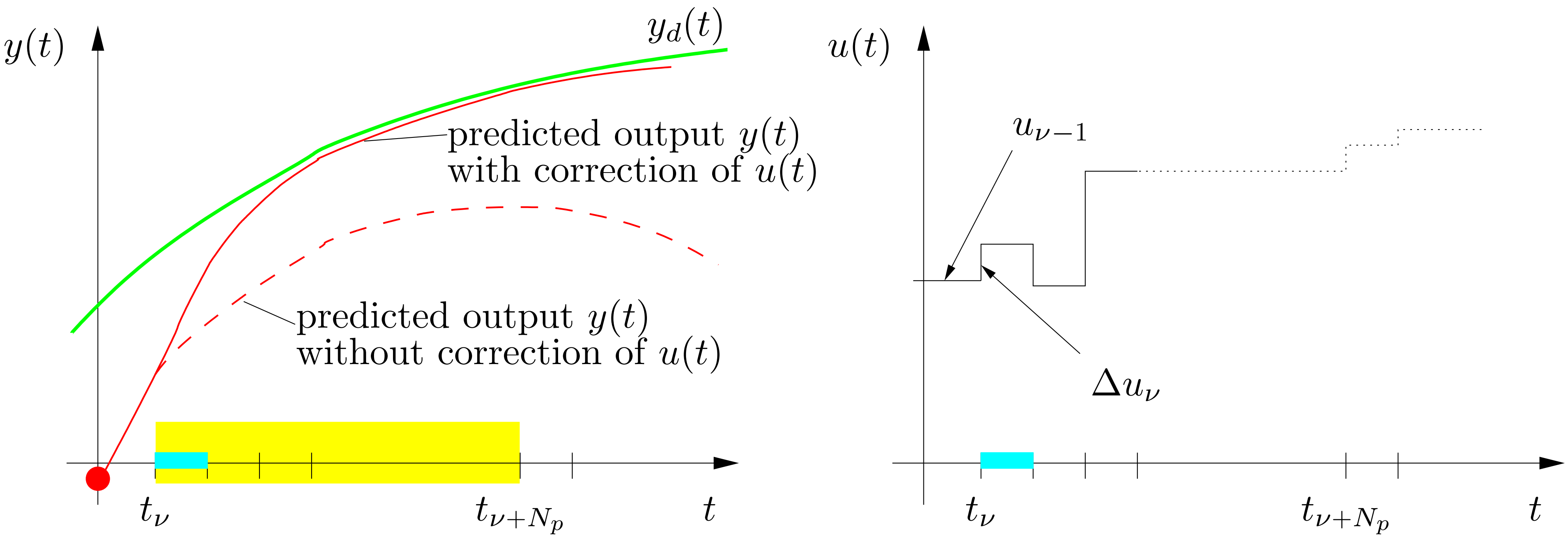
The usage of algorithmic differentiation allows for direct treatment of nonlinear system models.

Sensitivity-Based Model-Predictive Control





Sensitivity-Based Model-Predictive Control



- Sensitivity analysis for both analysis and design of control laws
- Consider a finite-dimensional dynamical system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \xi)$ with the state vector $\mathbf{x} \in \mathbb{R}^{n_x}$ (including observer state variables) and the parameter vector $\xi \in \mathbb{R}^{n_\xi}$ (including the system parameters $\mathbf{p}$ and the control inputs $\mathbf{u}$)

Compute piecewise constant control inputs $\mathbf{u}(t)$ for each time interval $t \in [t_\nu \,;\, t_{\nu+1})$, $0 \leq t_\nu < t_{\nu+1}$.

Sensitivity Analysis of Dynamical Systems

- Sensitivity of the solution $\mathbf{x}(t)$ to the set of ordinary differential equations $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \xi)$ with respect to a **time-invariant parameter vector** ξ

$$\frac{d}{dt}\left(\frac{\partial \mathbf{x}(t)}{\partial \xi_i}\right) = \frac{\partial \mathbf{f}(\mathbf{x}(t), \xi)}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}(t)}{\partial \xi_i} + \frac{\partial \mathbf{f}(\mathbf{x}(t), \xi)}{\partial \xi_i}$$

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- New state vectors ($\mathbf{x} \in \mathbb{R}^{n_x}$, $\xi \in \mathbb{R}^{n_\xi}$)

$$\mathbf{s}_i(t) := \frac{\partial \mathbf{x}(t)}{\partial \xi_i} \in \mathbb{R}^{n_x} \quad \text{for all} \quad i = 1, \ldots, n_\xi$$

$$\dot{\mathbf{s}}_i(t) = \frac{\partial \mathbf{f}(\mathbf{x}(t), \xi)}{\partial \mathbf{x}} \cdot \mathbf{s}_i(t) + \frac{\partial \mathbf{f}(\mathbf{x}(t), \xi)}{\partial \xi_i}$$

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- Initial conditions

$$\mathbf{s}_i(0) = \frac{\partial \mathbf{x}(0, \mathbf{p})}{\partial \xi_i} \quad \text{with} \quad \mathbf{s}_i(0) = 0 \quad \text{if } \mathbf{x}(0) \text{ is independent of } \xi_i$$

Sensitivity-Based Control Using Algorithmic Differentiation (1)

- Define the control error

$$J = \sum_{\mu=\nu}^{\nu+N_p} \mathcal{D}\left(\mathbf{y}\left(t_\mu\right) - \mathbf{y}_d\left(t_\mu\right)\right)$$

between the actual and desired system outputs $\mathbf{y}(t)$ and $\mathbf{y}_d(t)$, respectively, to achieve accurate tracking control behavior

- Define the output $\mathbf{y}(t)$ in terms of the state vector $\mathbf{x}(t)$ and the control $\mathbf{u}(t)$ (assumed to be piecewise constant for $t_\nu \leq t < t_{\nu+1}$) according to

$$\mathbf{y}(t) = \mathbf{g}\left(\mathbf{x}(t), \mathbf{u}(t)\right)$$

- Compute the differential sensitivity of J using algorithmic differentiation

Sensitivity-Based Control Using Algorithmic Differentiation (2)

- Correct the control input $\mathbf{u}(t_\nu)$ according to

$$\mathbf{u}(t_\nu) = \mathbf{u}(t_{\nu-1}) + \Delta\mathbf{u}_\nu \quad \text{with} \quad \Delta\mathbf{u}_\nu = -\left(\frac{\partial J}{\partial \Delta\mathbf{u}_\nu}\right)^{+} \cdot J \ ,$$

where $\mathbf{M}^{+} := \left(\mathbf{M}^T\mathbf{M}\right)^{-1}\mathbf{M}^T$ is the left pseudo-inverse of $\mathbf{M}$

- Compute the differential sensitivity of the error measure J

$$\frac{\partial J}{\partial \Delta\mathbf{u}_\nu} = \sum_{\mu=\nu}^{\nu+N_p} \left(\frac{\partial \mathcal{D}\left(\mathbf{g}(\mathbf{x},\mathbf{u}) - \mathbf{y}_d(t_\mu)\right)}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}(t_\mu)}{\partial \Delta\mathbf{u}_\nu} + \frac{\partial \mathcal{D}\left(\mathbf{g}(\mathbf{x},\mathbf{u}) - \mathbf{y}_d(t_\mu)\right)}{\partial \Delta\mathbf{u}_\nu} \right)$$

with the property

$$\frac{\partial \mathbf{x}(t_{\nu-1})}{\partial \Delta\mathbf{u}_\nu} = 0$$

- Evaluate $\frac{\partial \mathbf{g}}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{g}}{\partial \Delta\mathbf{u}_\nu}$ for $\mathbf{x} = \mathbf{x}(t_\mu)$ and $\mathbf{u} = \mathbf{u}(t_{\nu-1}) + \Delta\mathbf{u}_\nu$, $\Delta\mathbf{u}_\nu = 0$

Extensions to Sensitivity-Based Control of Uncertain Systems — Algorithm

Stage 1:

- Allow for uncertainty in parameters and measurements
- Enclose time discretization errors in the computation of the control input

$$\mathbf{u}(t_\nu) = \mathbf{u}(t_{\nu-1}) + \Delta\mathbf{u}_\nu \quad \text{with} \quad \Delta\mathbf{u}_\nu = -\sup\left(\left(\frac{\partial [J]}{\partial \Delta\mathbf{u}_\nu}\right)^+ \cdot [J]\right)$$

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Stage 2: Check for admissibility of the resulting solution with respect to state and input constraints

Stage 3: Adjust the control input if necessary according to worst-case overshoot

$$\overline{\Delta\mathbf{y}_\nu} := \max_{t\in\left[t_\nu\,;\,t_{\nu+\tilde{N}_p}\right]} \left\{0\,;\ \sup\left([\mathbf{y}(t)] - \mathbf{y}_d(t)\right)\right\}$$

Extensions to Sensitivity-Based Control of Uncertain Systems — Example (1)

- Control of a double integrating plant

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ F_d \end{bmatrix} \quad \text{with} \quad m \in [0.9\ ;\ 1.1]\ ,\ \ F_d \in [-0.1\ ;\ 0.1]$$

- Definition of the desired output trajectory

$$y_d(t) = x_{1,d}(t) = 1 - e^{-t}$$

with the inconsistent initial state $\mathbf{x}(0) = \begin{bmatrix} -1 & 0 \end{bmatrix}^T$

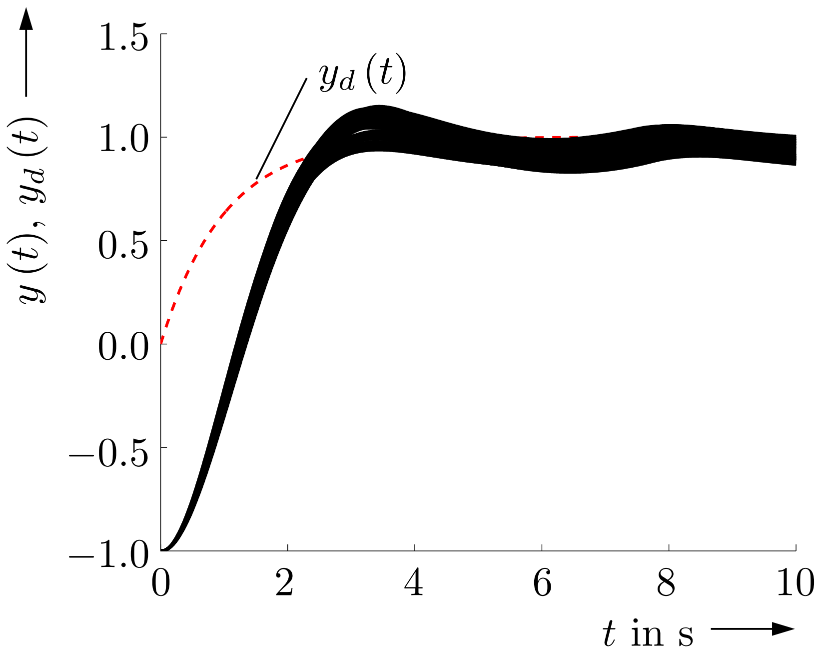
- **Direct computation of a piecewise constant control** with a time-invariant step size $t_{\nu+1} - t_\nu = 0.01$ and $N = 200$

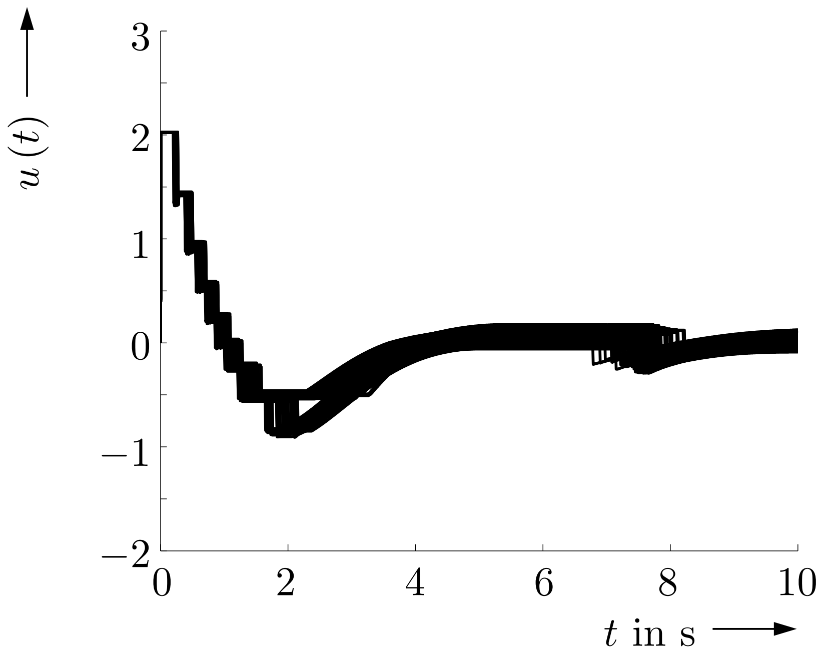
- Guaranteed admissibility of the solution in spite of bounded measurement errors

$$x_1(t) \in x_{1,m}(t) + [-0.01\ ;\ 0.01] \qquad x_2(t) \in x_{2,m}(t) + [-0.01\ ;\ 0.01]$$

Extensions to Sensitivity-Based Control of Uncertain Systems — Example (2)

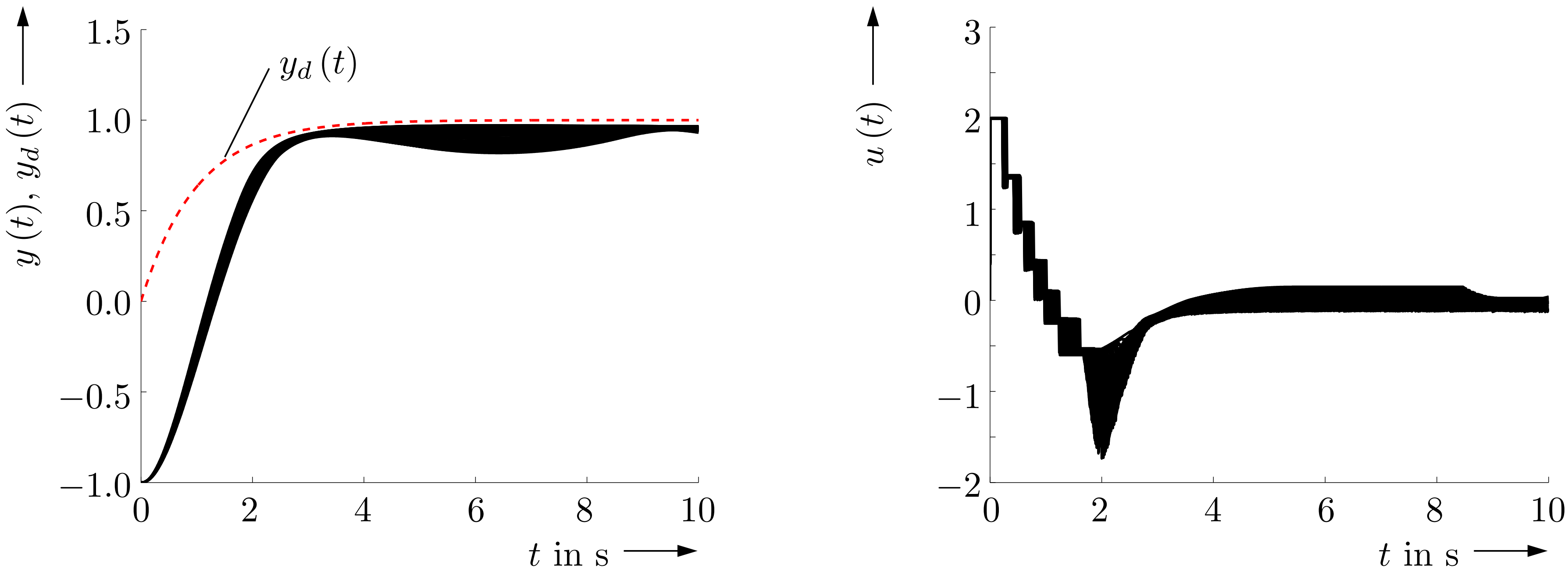
Result: Grid-based simulation of sensitivity-based approach without guaranteed overshoot prevention





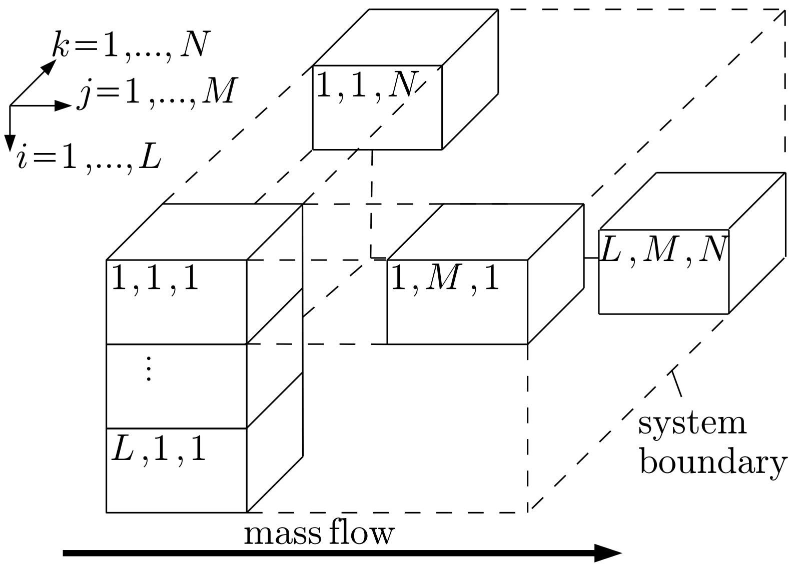
Extensions to Sensitivity-Based Control of Uncertain Systems — Example (4)

Result: Grid-based validation of sensitivity-based approach with guaranteed overshoot prevention



Practical Application Scenario: Temperature Control for Solid Oxide Fuel Cell Systems (1)

- Control-oriented thermal SOFC model: Semi-discretization into $n_x = L \cdot M \cdot N$ finite volume elements



- Introduction of the state vector $\mathbf{x}^T = [\vartheta_{1,1,1}, ..., \vartheta_{L,M,N}] \in \mathbb{R}^{n_x}$ (piecewise homogeneous temperature values)

- Restriction to the configurations $L = M = N = 1$ and $L = 1$, $M = 3$, $N = 1$

Practical Application Scenario: Temperature Control for Solid Oxide Fuel Cell Systems (2)

- Design of a predictive control procedure such that
  - System inputs and operating temperature stay close to the desired set-point
  - Large spatial gradients of the temperature distribution are penalized
  - Local violations of the maximum admissible cell temperature are prevented with certainty (in a rigorous formulation)
  - Temporal variation rates of the physical system inputs do not violate given bounds (in a weak formulation)

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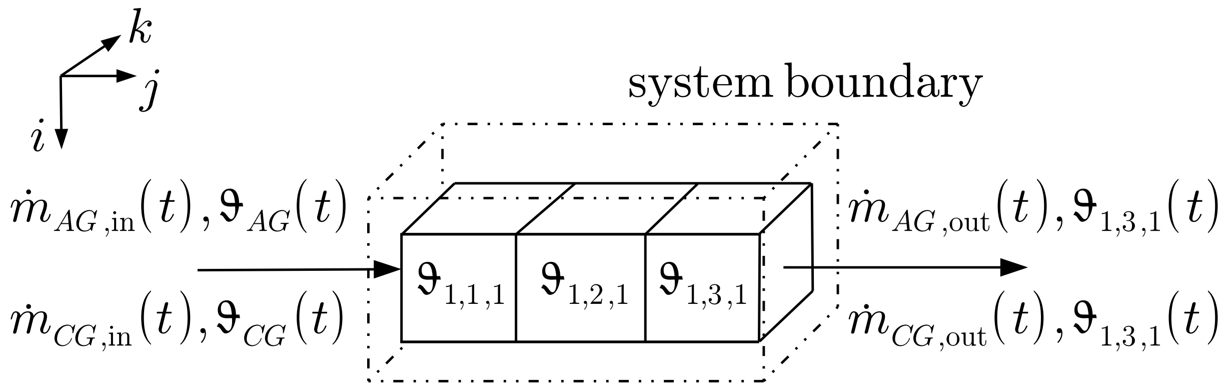
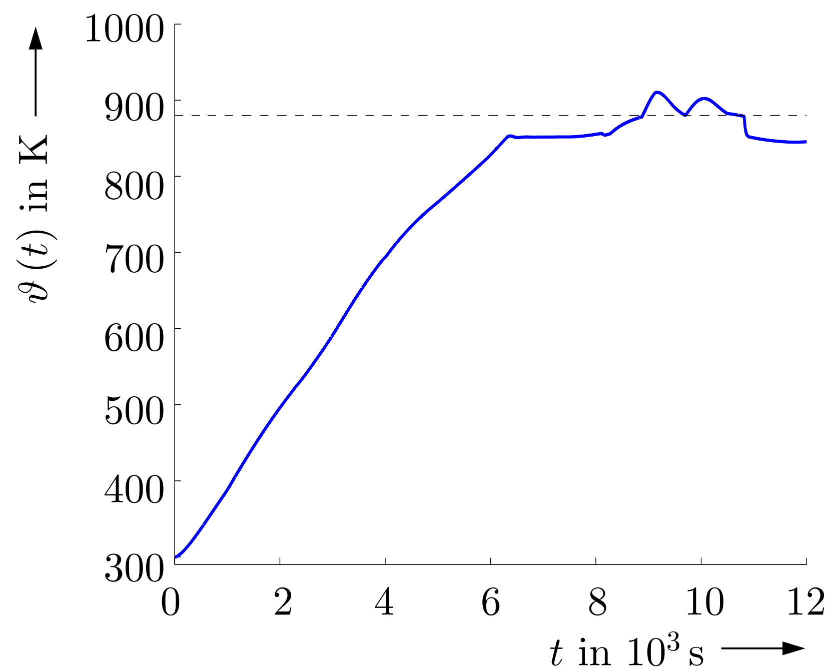


Interval-Based Predictive Control (1)

Result: Cell temperature for the scalar system model (desired operating temperature: $850\,\mathrm{K}$, max. admissible temperature $880\,\mathrm{K}$ with varying properties of the anode gas and the electric load)

without overshoot prevention



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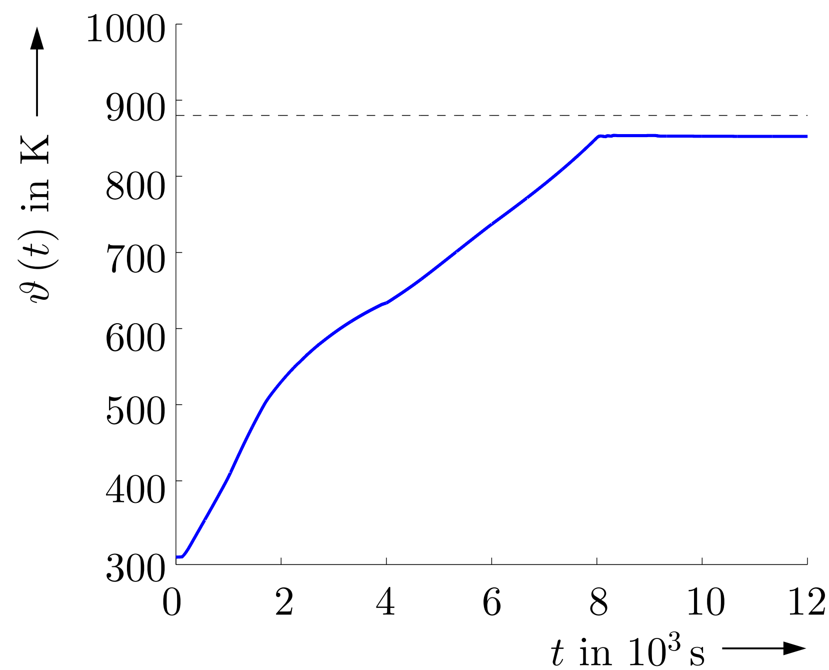

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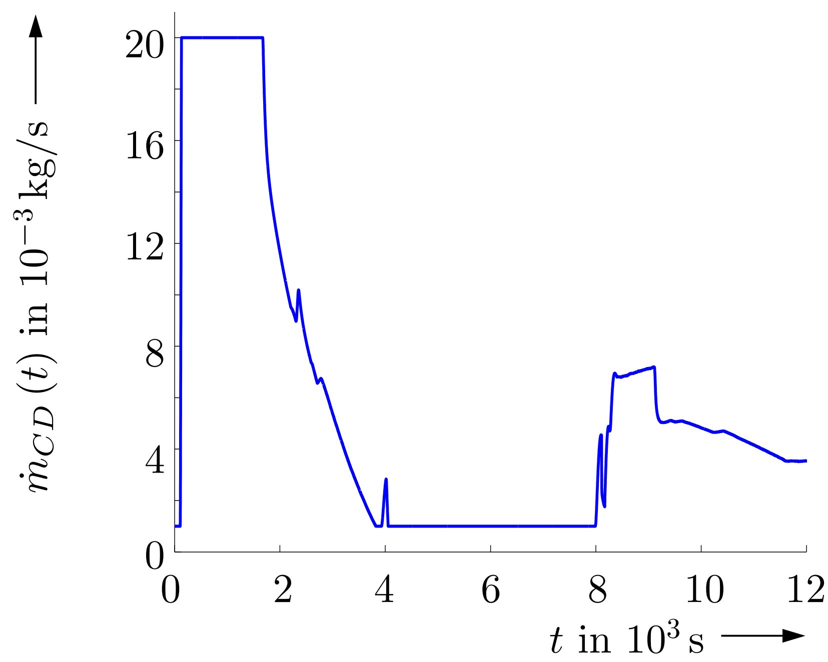
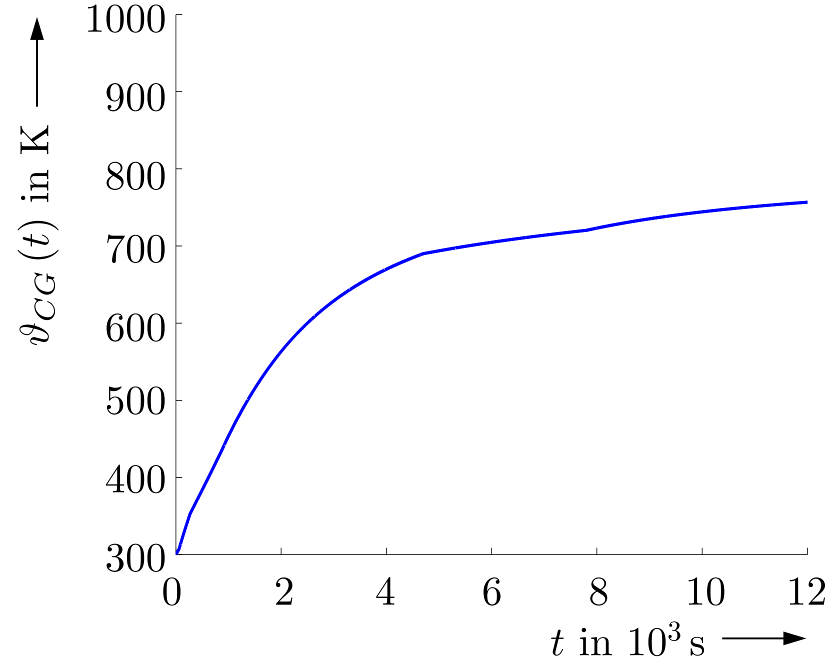
mass flow of cathode gas



preheater temperature



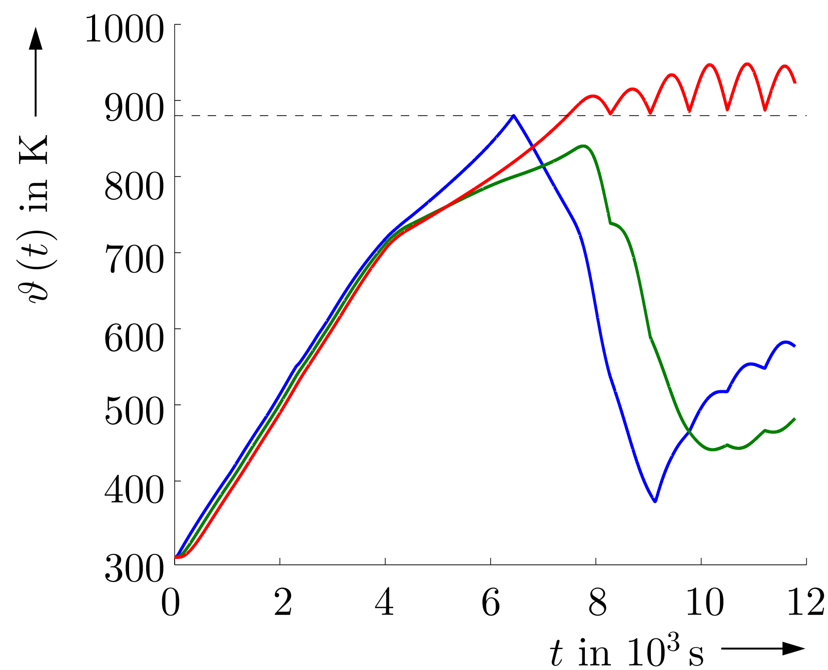
Interval-Based Predictive Control (3)

Result: Cell temperature for the system model with $n_x = 3$ states (desired operating temperature: $850\,\mathrm{K}$, max. admissible temperature $880\,\mathrm{K}$ with varying properties of the anode gas and the electric load)

Undesirable behavior after $t = 11,000\,\mathrm{s}$ can be predicted from simulations and avoided by a suitable supervisory control for the remaining system inputs

without overshoot prevention

$\vartheta_{1,1,1}$, $\vartheta_{1,2,1}$, $\vartheta_{1,3,1}$



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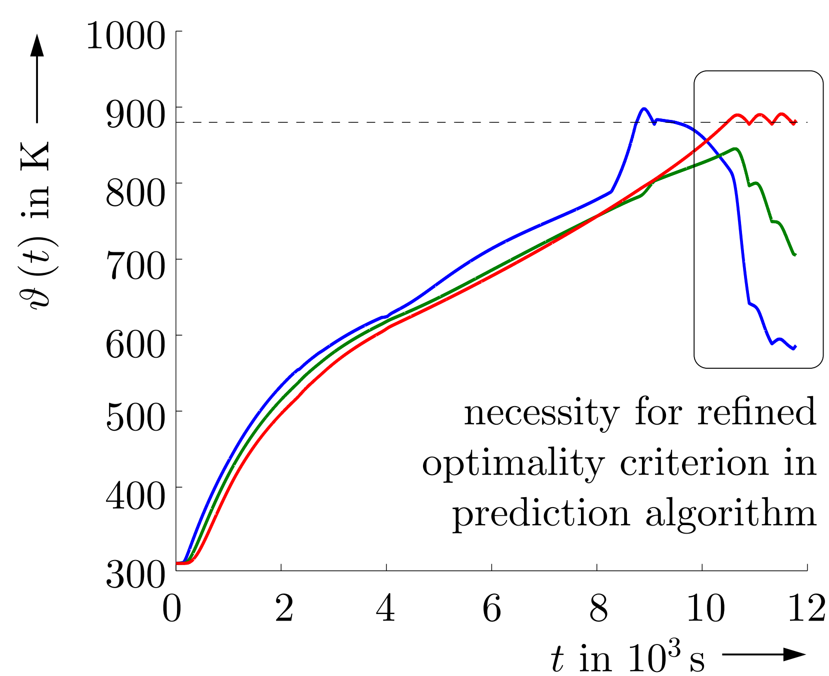

Derivation of a Physically-Motivated Criterion for the Detection and Reduction of Overestimation (1)

- Prediction of the stack temperatures over the time horizon $t \in \left[t_\nu \; ; \; t_{\nu+N_p}\right]$ with $N_p > 0$ steps and constant sampling time $T := t_{\nu+1} - t_\nu$

$\Longrightarrow$ Overestimation in the state enclosures can make the predictive control procedure inefficient

- Energy-related criterion for the **detection of overestimation**
- **Variant 1:** Direct evaluation of

$$E_\mu := E\left(t_\mu\right) = \sum_{i,j,k} c_{i,j,k} \cdot m_{i,j,k} \cdot \vartheta_{i,j,k}(t_\mu)$$

- **Variant 2:** Integral formulation (with typically tighter bounds)

$$E_\mu = E_\nu + \int_{t_\nu}^{t_\mu} \dot{E}\left(\tau\right) d\tau = E_\nu + \int_{t_\nu}^{t_\mu} \left(\sum_{i,j,k} c_{i,j,k} \cdot m_{i,j,k} \cdot \dot{\vartheta}_{i,j,k}(\tau) \right) d\tau$$

Derivation of a Physically-Motivated Criterion for the Detection and Reduction of Overestimation (2)

- Simplification for state-independent and time-invariant parameters $c_{i,j,k}$ and $m_{i,j,k}$ which are identical for all finite volume elements
- Modified formulation
  - **Variant 1:** Direct evaluation of

$$E_\mu := E\left(t_\mu\right) = \sum_{i,j,k} \vartheta_{i,j,k}(t_\mu)$$

  - **Variant 2:** Integral formulation

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- Determine the offset $E_\nu \in [E_\nu]$ on the basis of measured temperatures (including measurement tolerances and estimation errors)

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Reduced overestimation on **variant 2** since the heat flow over boundaries between neighboring finite volume elements cancels out exactly (energy conservation: first law of thermodynamics!)

Discrete-Time Formulation of the Predictive Control Algorithm (1)

- Determine state enclosure for $t = t_\nu$: $\vartheta_{i,j,k}(t_\nu) \in [\vartheta_{i,j,k}(t_\nu)]$
- Discrete-time evaluation of the state equations over the complete prediction horizon $\left[t_\nu \,;\, t_{\nu+N_p}\right]$, $\mu > \nu$

$$\vartheta_{i,j,k}(t_\mu) \in [\vartheta_{i,j,k}(t_{\mu-1})] + T \cdot \left[\dot{\vartheta}_{i,j,k}(t_{\mu-1})\right] \quad \text{with} \quad \mathbf{u} = \mathbf{u}(t_{\nu-1})$$

- Simultaneous evaluation of the performance criterion
- Evaluation of the corresponding sensitivities by means of algorithmic differentiation
- Overestimation criteria

$$E_\mu \in \sum_{i,j,k} [\vartheta_{i,j,k}(t_\mu)] \quad \text{and} \quad E_\mu \in \left[\tilde{E}_\mu\right] := [E_\nu] + \sum_{\mu'=\nu}^{\mu} \left(\sum_{i,j,k} \left[\dot{\vartheta}_{i,j,k}(t'_\mu)\right] \right)$$

Discrete-Time Formulation of the Predictive Control Algorithm (2)

- Reduction of the conservativeness with respect to the maximum predicted temperature for all $t \in \left[t_\nu \; ; \; t_{\nu+N_p}\right]$ by the following consistency test
  - Subdivide $[\vartheta_{i,j,k}(t_\mu)]$ into subintervals $\left[\vartheta'_{i,j,k}(t_\mu)\right]$ along the longest edge
  - Evaluate

$$E'_\mu \in [E'_\mu] = \sum_{i,j,k} [\vartheta'_{i,j,k}(t_\mu)]$$

    for all subintervals of the predicted state enclosure $[\vartheta_{i,j,k}(t_\mu)]$

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  - Evaluate
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  for all subintervals of the predicted state enclosure $[\vartheta_{i,j,k}(t_{\mu})]$
- Classification of the resulting subintervals
  - Guaranteed caused by overestimation if $\left[E'_{\mu}\right] \cap \left[\tilde{E}_{\mu}\right] = \emptyset$
  - Undecided for $\left[E'_{\mu}\right] \cap \left[\tilde{E}_{\mu}\right] \neq \emptyset$ and $\left[E'_{\mu}\right] \not\subseteq \left[\tilde{E}_{\mu}\right]$
  - Consistent for $\left[E'_{\mu}\right] \subseteq \left[\tilde{E}_{\mu}\right]$, where $\left[\tilde{E}_{\mu}\right]$ denotes the result of variant 2

Discrete-Time Formulation of the Predictive Control Algorithm (2)

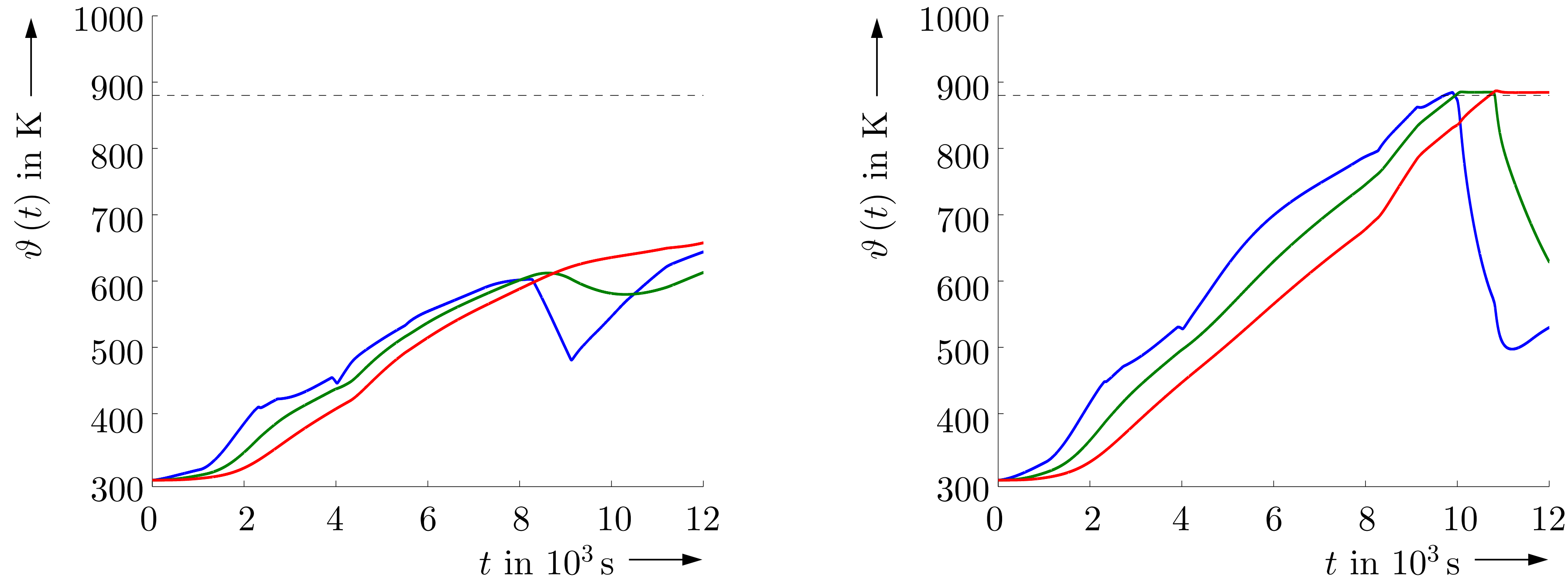
- Reduction of the conservativeness with respect to the maximum predicted temperature for all $t \in \left[t_{\nu}\ ;\ t_{\nu+N_p}\right]$ by the following consistency test
  - Subdivide $[\vartheta_{i,j,k}(t_{\mu})]$ into subintervals $\left[\vartheta'_{i,j,k}(t_{\mu})\right]$ along the longest edge
  - Evaluate
$$E'_{\mu} \in [E'_{\mu}] = \sum_{i,j,k} [\vartheta'_{i,j,k}(t_{\mu})]$$
    for all subintervals of the predicted state enclosure $[\vartheta_{i,j,k}(t_{\mu})]$
- Classification of the resulting subintervals
  - Guaranteed caused by overestimation if $[E'_{\mu}] \cap \left[\tilde{E}_{\mu}\right] = \emptyset$
  - Undecided for $[E'_{\mu}] \cap \left[\tilde{E}_{\mu}\right] \neq \emptyset$ and $[E'_{\mu}] \not\subseteq \left[\tilde{E}_{\mu}\right]$
  - Consistent for $[E'_{\mu}] \subseteq \left[\tilde{E}_{\mu}\right]$, where $\left[\tilde{E}_{\mu}\right]$ denotes the result of variant 2
- Re-evaluate $[J]$ for the reduced predicted overshoot
  $\Longrightarrow$ Perform the sensitivity-based control update as for the illustrative example

Interval-Based Predictive Control: Results (cont'd)

Result: Cell temperature for the system model with $n_x = 3$ states (desired operating temperature: $850\,\mathrm{K}$, max. admissible temperature $880\,\mathrm{K}$ with varying properties of the anode gas and the electric load)

additional penalization of temperature gradients

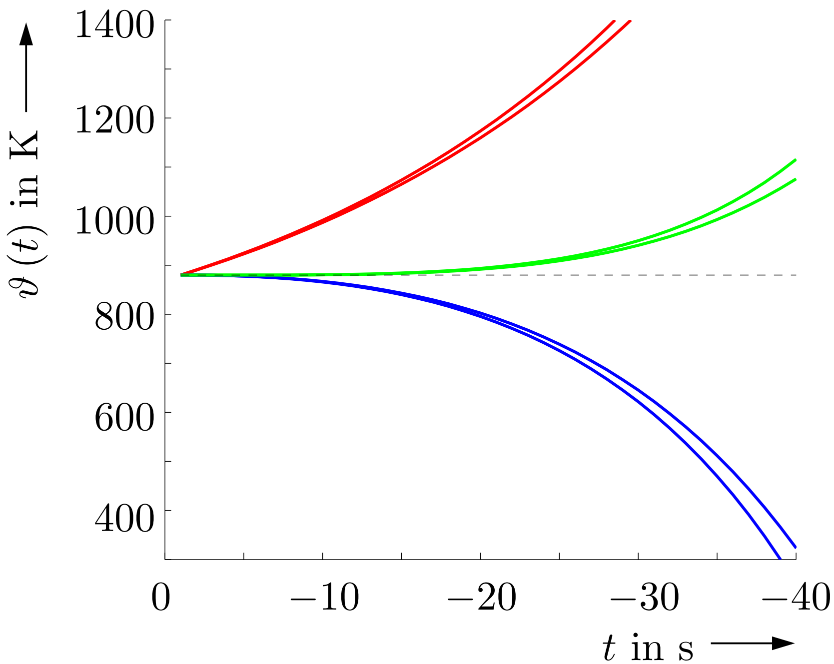
penalization of temperature gradients and extension by consistency test



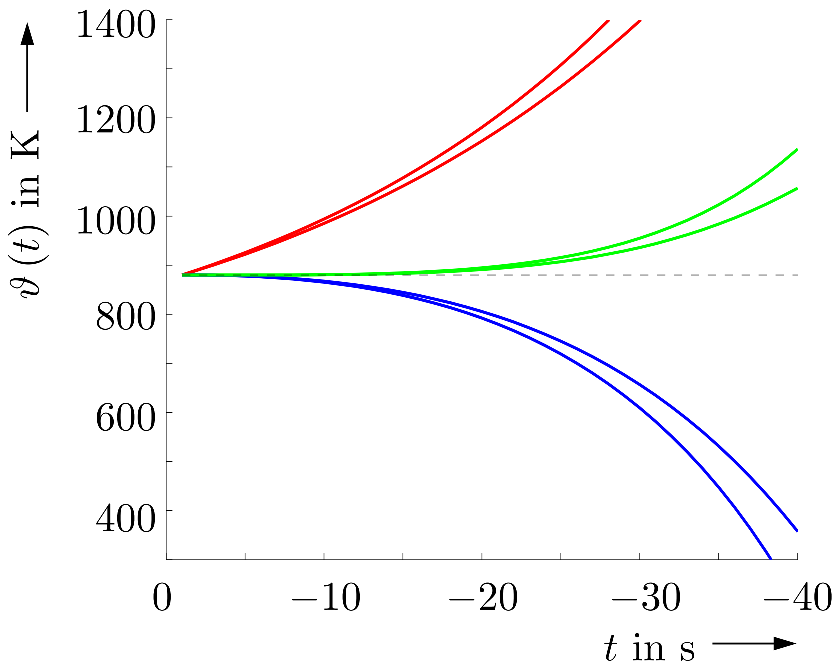
Controllability Analysis by Backward Evaluation (1)

Backward integration for the offline detection of admissible operating regions for the cell temperature

0.5% *uncertainty*



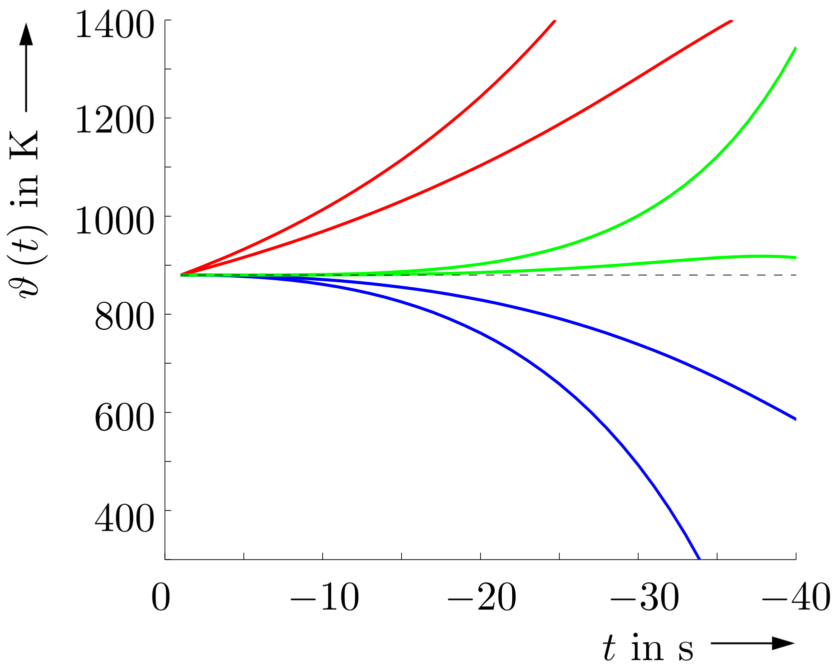
1.0% *uncertainty*



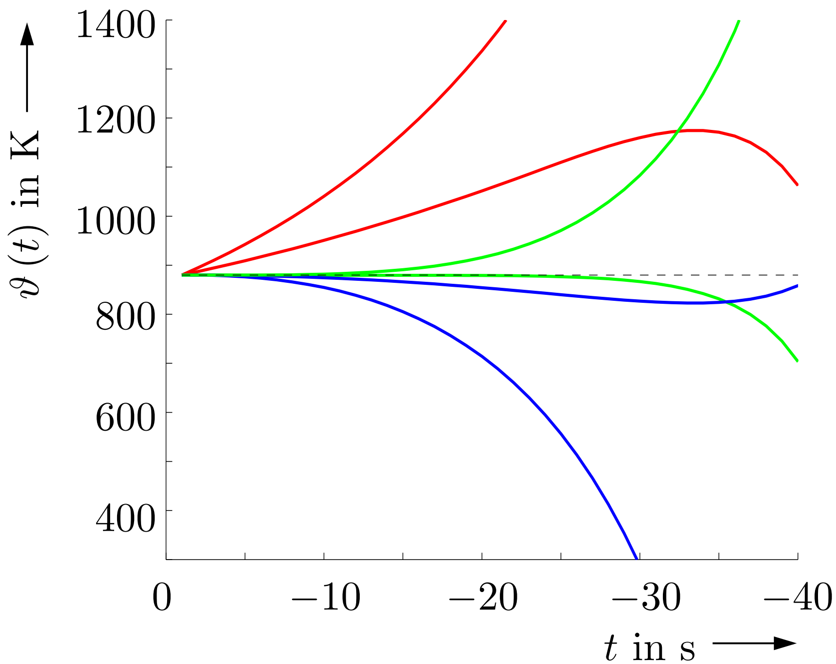
Controllability Analysis by Backward Evaluation (2)

Backward integration for the offline detection of admissible operating regions for the cell temperature

5.0% *uncertainty*



10.0% *uncertainty*



Sensitivity-Based State and Parameter Identification (1)

- General (vector-valued) performance criterion

$$\mathbf{J} = \sum_{i=k-N}^{k} \mathcal{D}\left(\hat{\mathbf{y}}(t_i) - \mathbf{y}_m(t_i)\right)$$

- Definition of a quadratic error measure

$$\mathcal{D}_\mu = \left(\hat{\mathbf{y}}(t_i) - \mathbf{y}_m(t_i)\right)^T \mathbf{P}_\mu \left(\hat{\mathbf{y}}(t_i) - \mathbf{y}_m(t_i)\right)$$

- Main challenge: Nonlinear dependency on optimization variables with a large number of measured data points N
- Underlying evaluation of the state equations: Explicit Euler method

$$\hat{\mathbf{x}}(t_j) = \hat{\mathbf{x}}(t_{k-N}) + \sum_{i=k-N}^{j-1} \Delta T \cdot \mathbf{f}(\hat{\mathbf{x}}(t_i), \mathbf{u}(t_i))$$

Sensitivity-Based State and Parameter Identification (2)

- Output equation

$$\hat{\mathbf{y}}\left(t\right)=\mathbf{g}\left(\hat{\mathbf{x}}\left(t\right),\mathbf{u}\left(t\right)\right)\quad,\quad\hat{\mathbf{y}}\in\mathbb{R}^{n_y}$$

- Sensitivity analysis for the gradient-like procedure

$$\frac{\partial J_\mu}{\partial\hat{\mathbf{x}}\left(t_{k-N}\right)}=2\sum_{i=k-N}^{k}\left\{\left(\frac{\partial\hat{\mathbf{x}}\left(t_i\right)}{\partial\hat{\mathbf{x}}\left(t_{k-N}\right)}\right)^T\right.$$
$$\left.\left(\frac{\partial\mathbf{g}\left(\hat{\mathbf{x}}\left(t_i\right),\mathbf{u}\left(t_i\right)\right)}{\partial\hat{\mathbf{x}}\left(t_i\right)}\right)^T\mathbf{P}_\mu\left(\hat{\mathbf{y}}\left(t_i\right)-\mathbf{y}_m\left(t_i\right)\right)\right\}$$

- Correction step

$$\Delta\hat{\mathbf{x}}(t_{k-N})=-\alpha\left(\frac{\partial\mathbf{J}}{\partial\hat{\mathbf{x}}(t_{k-N})}\right)^+\cdot\mathbf{J}$$

with $\hat{\tilde{\mathbf{x}}}\left(t_{k-N}\right)=\hat{\mathbf{x}}\left(t_{k-N}\right)+\Delta\hat{\mathbf{x}}\left(t_{k-N}\right)$ and the optional step-size factor α

Sensitivity-Based State and Parameter Identification (3)

- Newton-like procedure: Second-order approximation of $\mathbf{J}$

$$J_\mu\left(\hat{\mathbf{x}}+\Delta\hat{\mathbf{x}}_\mu\right) \approx J_\mu\left(\hat{\mathbf{x}}\right) + \left.\frac{\partial J_\mu}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}} \Delta\hat{\mathbf{x}}_\mu \quad + \frac{1}{2}\Delta\hat{\mathbf{x}}_\mu^T \left.\frac{\partial^2 J_\mu}{\partial \mathbf{x}^2}\right|_{\mathbf{x}=\hat{\mathbf{x}}} \Delta\hat{\mathbf{x}}_\mu$$

- Update rule

$$\Delta\hat{\mathbf{x}}_\mu(t_{k-N}) := -\left(\left.\frac{\partial^2 J_\mu}{\partial \mathbf{x}^2}\right|_{\mathbf{x}=\hat{\mathbf{x}}}\right)^+ \left.\frac{\partial J_\mu}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}}$$

A. Rauh, L. Senkel, H. Aschemann: *Sensitivity-Based State and Parameter Estimation for Fuel Cell Systems*, Proc. of 7th IFAC Symposium on Robust Control Design, Aalborg, Denmark, 2012.

$\Longrightarrow$ Guaranteed proof of stability by procedure similar to verification of sliding mode state estimation:

A. Rauh, L. Senkel, H. Aschemann: *Interval-Based Sliding-Mode Observer Design for Nonlinear Systems with Bounded Uncertainties*, in preparation for ECC 2013, Zurich, Switzerland.

Conclusions and Outlook on Future Work

- Framework for sensitivity-based open-loop and closed-loop control with real-life applications
- Extension of sensitivity-based control to systems with interval uncertainties
  $\Longrightarrow$ Guarantee the compliance with state and control constraints
- Development of a general framework for interval arithmetic, sensitivity-based model-predictive control
  $\Longrightarrow$ Problem-dependent definition of corresponding cost functions

Conclusions and Outlook on Future Work

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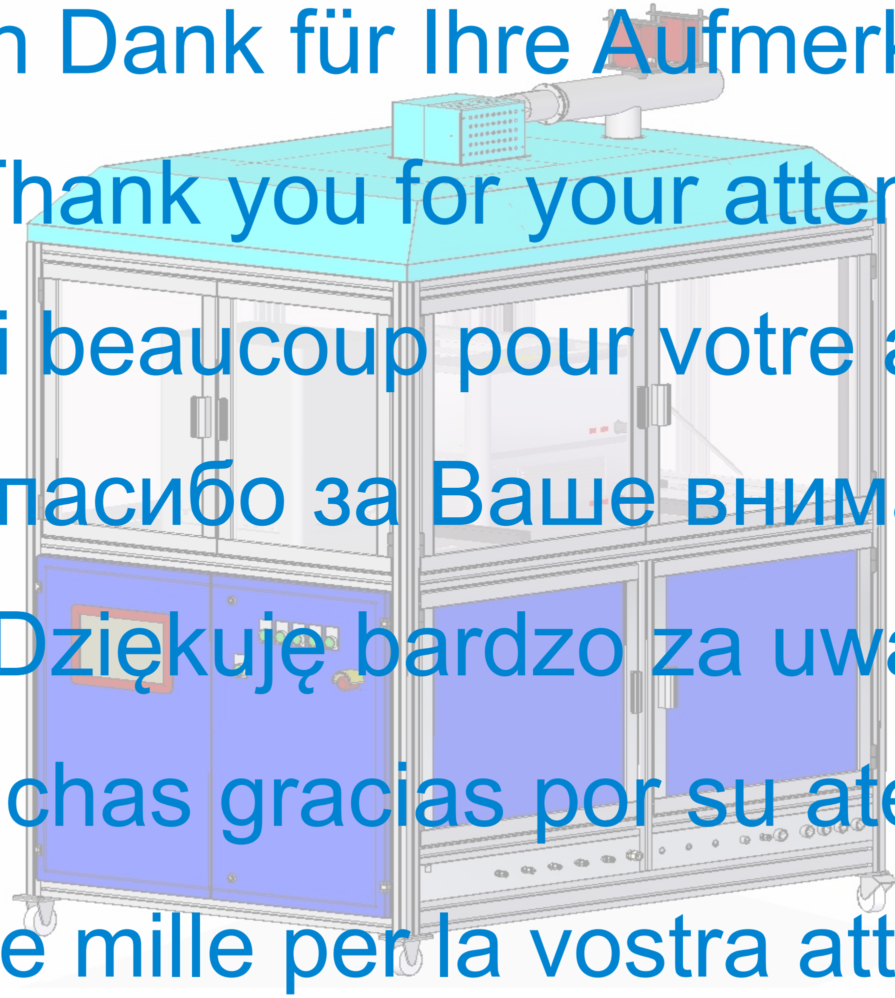
  $\Longrightarrow$ Guarantee the compliance with state and control constraints
- Development of a general framework for interval arithmetic, sensitivity-based model-predictive control
  $\Longrightarrow$ Problem-dependent definition of corresponding cost functions
- Extension of sensitivity-based control to state and disturbance estimation (duality of control and observer synthesis)
- Verification of (asymptotic) stability
- Gain scheduling for sliding mode control with interval uncertainties



Vielen Dank für Ihre Aufmerksamkeit!

Thank you for your attention!

Merci beaucoup pour votre attention!

Спасибо за Ваше внимание!

Dziękuję bardzo za uwagę!

¡muchas gracias por su atención!

Grazie mille per la vostra attenzione!