The Problem about Resistant Force for a Cylindrical Striker and Composite Target

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A *low-speed impact penetration has been considered. The model of incompressible rigid plastic material is used. The analytical solution of the problem is based on the theorem about upper bound limit load. Analytical formulae are used to get the resistant force acting on penetrating striker.* *Keywords - Striker, perfectly plastic medium, incompressible, velocity field, upper limit load*

It is proposed to use a known method of solving problems of plasticity theory which is based on one theorem of bound analysis - the theorem of upper bound of limit load. This method is relatively simple and it makes possible to obtain a quantitative solution, when the Saint-Venant model is used. This model supposes the material of the target to be incompressible and described by ideal plastic medium. The essence of this method is the use of the principal energy equality - equilibrium equation in the integral Lagrange form.

A key aspect of the use of this equation is assignment of kinematical possible velocity fields. The main difficulties of the problem solution are connected with assigning of kinematic possible flows for two component material of the composite target – layer and foundation. The fields have to satisfy the incompressibility equation in both areas and condition of continuity for normal components of velocities along boundaries between the areas. The specified velocity fields makes it possible to calculate the unknown vector of surfaces forces, the above theorem being used to estimate the upper bound of the actual load. The more accurately the flow field is assigned the more accurately limit load will be calculated from Lagrange's equation.

The admissible velocity fields are assumed. The fields allow calculating all integrals analytically and getting a relatively simple calculation of the resistant force.
Smooth cylindrical striker with radius of a and initial velocity of V_0 impacts a composite target – layer on semi space. As a result of the impact a channel of the flow forms (Fig.1).

H - thickness of the layer. a – radius of the striker; b – radius of the flow channel.

t – distance between the cylinder and channel bottom; t_1 – distance of lines 3 and 4 intersection from the channel bottom.

The lines of velocities discontinuities are numbered from 1 till 5 (Fig.2).

Zone 1 and Zone 2 – a particle of the target rotates and moves towards free surface; Zone 3 – vertical flow; Zone 4 - "dead" zone.

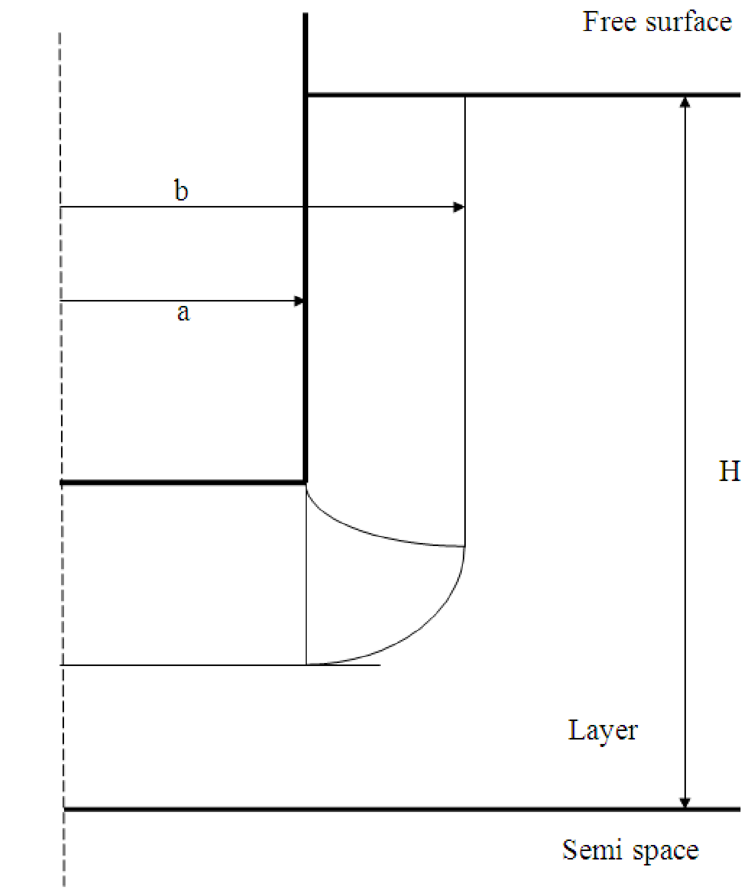
Free surface
b
a
H
Layer
Semi space

Fig.1

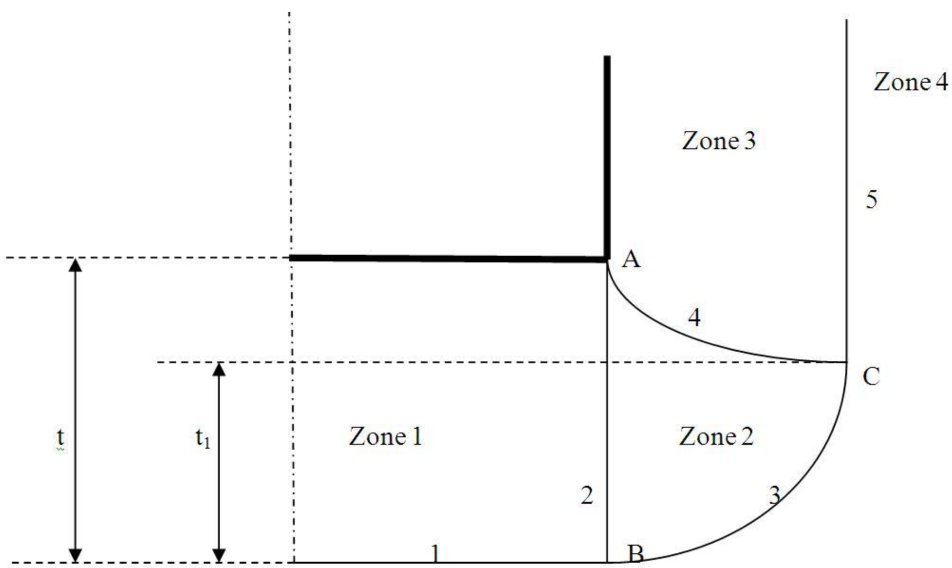
Zone 4
Zone 3
5
A
4
C
t
t1
Zone 1
Zone 2
2
3
1
B

Fig.2

Follow В.М.Фомин, А.И.Гулидов, Г.А.Сапожников и др. (1999) and Babakov (2009) we assume next velocity fields

Zone 1:

This zone is a disk deforming plastically. The velocity component v_z obeys the boundary conditions on the punch and on the bottom of flow channel. It is not so difficult to see that flow field may be described by the simple formulae

$$v_r^{(1)} = V_0 \frac{r}{2t}, \qquad v_z^{(1)} = -V_0 \frac{z}{t}$$

where V_0 is the velocity of the cylinder; it is possible to put this value equal to 1 for simplicity, so we can write

$$v_r^{(1)} = \frac{r}{2t}, \qquad v_z^{(1)} = -\frac{z}{t}$$

Zone 3:

Zone 3 is the zone of tube flow. The velocity of Zone 3 is described by a rigid body motion in the direction of the axis of symmetry, denote by V_1

$$v_r^{(3)} = 0, \qquad v_z^{(3)} = V_1 = \frac{1}{\epsilon^2 - 1}$$

where $\epsilon = \frac{b}{a}$. The relation for $v_z^{(3)}$ has been got directly from condition of incompressibility, i.e. volume constancy.

Zone 2:

Zone 2 is the zone of plastic deformations. The components of velocity vector are assumed

$$v_r^{(2)} = \frac{a^2}{2t}\frac{1}{r}, \qquad v_z^{(2)} = \frac{t_1}{t}\frac{1}{\epsilon^2 - 1}$$

The zone is separated from the zone 1 by the surface AB, from the zone 3 by the surface AC and from the zone 4 by the surface BC. These surfaces are the surfaces of velocity discontinuity but normal velocity component must be continuous on the passage over these surfaces. Notice that surfaces AB and AC are unknown.

Zone 4: $\vec{v}_4 = 0,$ it means that Zone 4 is dead zone.

Notice it is not hard to show that the supposed fields are kinematic admissible, since these satisfy the incompressibility equation

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0,$$

boundary conditions and the continuity conditions on any surface. The behavior of velocity components on surfaces AC and BC requires special consideration which will make possible to determine the surfaces AC and BC.

The equilibrium equation in Lagrange form for Saint-Venant model is

$$\int_S \sigma_{ni}\, v_i dS = \tau_s \int_V HdV + \sum_{k=1}^{N} \tau_s \int_{L_k} |[v_\tau]| dL_k$$

where $H = \sqrt{2\epsilon_{ij}\epsilon_{ij}}$, $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$

v_i components of kinematic admissible velocity field;
L_k surfaces of discontinuity of velocity;
$[v_\tau]$ jump of shear velocity.
τ_s limit of plasticity
Substituting the components of the velocity vector

$$\epsilon_r^{(1)} = \frac{1}{2t}\ , \qquad \epsilon_\varphi^{(1)} = \frac{1}{2t}\ , \qquad \epsilon_z^{(1)} = -\frac{1}{t}\ ,$$

$$\epsilon_r^{(2)} = -\frac{a^2}{2tr^2}\ , \qquad \epsilon_\varphi^{(2)} = \frac{a^2}{2tr^2}\ , \qquad \epsilon_z^{(2)} = 0\ ,$$

$$\epsilon_{ij}{}^{(3)} = 0 \quad .$$

Therefore the values H_i are equal respectively to

$$H_1 = \frac{\sqrt{3}}{t}\ , \quad H_2 = \frac{a^2}{tr^2}\ , \quad H_3 = 0$$

The total process of the penetration consists of four cases that depend on depth of penetration. Boundary between layer and semi space can:

1. be below flow channel;
2. cut BC surface;
3. cut AC surface;
4. above bottom of the striker.

Further we demonstrate realizing of the method on the case 3 when boundary cuts AC.

This is the third case that can be described by next inequalities:

$$t - t_1 > H - h$$
$$H > h$$

h – depth of penetration (distance from free surface till bottom of the striker).

The equations of unknown surfaces AC and BC can be found from condition of continuum medium, i.e. from condition of continuity of normal velocity component on these surfaces AC and AB respectively [1]:

$$z = \frac{t - t_1}{a^2(\in^2 - 1)}(a^2 \in^2 - r^2) + t_1,$$

$$z = \frac{t_1}{\in^2 - 1}\left(\frac{r^2}{a^2} - 1\right).$$

Co-ordinates of point O (intersection boundary plane with AC) can be found with elementary consideration.

Plastic work in Zone 1 can be calculated by integration of the integral

$$A_1 = \int_0^{2\pi} \mathrm{d}\varphi \int_0^a (\tau_{s1} \int_{t+h-H}^{t} + \tau_{s2} \int_0^{t+h-H}) H_1 r dr dz =$$

$$= \frac{\pi a^2 \sqrt{3}}{t}[(H - h)(\tau_{s1} - +\tau_{s2}) + t\tau_{s2}].$$

Plastic work in Zone 2 consists of three integrals

$$A_{21} = \tau_{s1} \int_0^{2\pi} \mathrm{d}\varphi \int_a^{R_{01}} \int_{t+h-H}^{z_{C_4}} H_2 r dr dz$$

$$= \tau_{s1} \frac{\pi a^2}{t}\left[\frac{t - t_1}{\in^2 - 1}\left(1 - \frac{R_{01}^2}{a^2}\right) + \left(\frac{t - t_1}{\in^2 - 1} + H - h\right) ln \frac{R_{01}^2}{a^2}\right]$$

$$A_{22} = \tau_{s2} \int_0^{2\pi} \mathrm{d}\varphi \int_a^{R_{01}} \int_{z_{C_3}}^{t+h-H} \frac{a^2}{tr^2} r dr dz$$

$$= \tau_{s2} \frac{\pi a^2}{t}\left[\frac{t_1}{\in^2 - 1}\left(1 - \frac{R_{01}^2}{a^2}\right) + \left(\frac{t_1}{\in^2 - 1} + t + h - H\right) ln \frac{R_{01}^2}{a^2}\right]$$

$$A_{23} = \tau_{s2} \int_0^{2\pi} \mathrm{d}\varphi \int_{R_{01}}^{a\in} \int_{z_{C_3}}^{z_{C_4}} \frac{a^2}{tr^2} r dr dz$$

$$= \tau_{s2}\frac{\pi a^2}{t}\left[\frac{t}{\epsilon^2-1}\left(\frac{R_{01}^2}{a^2}-\epsilon^2\right)+\frac{t\,\epsilon^2}{\epsilon^2-1}ln\frac{R_{01}^2}{a^2}\right]$$

Where $\epsilon^2=\frac{b^2}{a^2}$ and R_{01} is r co-ordinate of point where boundary between layer and semi space cuts AC (curve C4). The value of R_{01} is expressed as

$$R_{01} = \sqrt{\frac{(H-h)(\epsilon^2-1)+t-t_1}{t-t_1}}$$

Now we need to calculate plastic work on the surfaces of velocity discontinuities (B_1– surface integral along the bottom of the channel; B_2– surface integral along AB; B_3– surface integral along BC; B_4– surface integral along AC; B_5– surface integral along the cylindrical surface towards free surface.

$$B_1=\tau_{s2}\frac{\pi a^3}{3t}$$

$$B_2 = \int_0^{2\pi} d\varphi \left(\tau_{s1}\int_{t+h-H}^{t} + \tau_{s2}\int_0^{t+h-H}\right)\left[\frac{z}{t}+\frac{t}{t_1(\epsilon^2-1)}\right]ad\varphi dz$$

$$= \frac{2\pi a}{t}\left[\frac{tt_1\tau_{s1}+t_1(t+h-H)(\tau_{s2}-\tau_{s1})}{\epsilon^2-1}+\frac{1}{2}\left(\tau_{s1}t^2+(t+h-H)^2(\tau_{s2}-\tau_{s1})\right)\right]$$

$$B_3=\tau_{s2}\frac{\pi a}{t}\left[a^2(\epsilon-1)+\frac{4t_1(\epsilon^3-1)}{\epsilon\,(\epsilon^2-1)^2}\right]$$

$$B_4 = \int_0^{2\pi} d\varphi \left(\tau_{s1}\int_a^{R_{01}} + \tau_{s2}\int_{R_{01}}^{a\epsilon}\right)\left[\frac{2(t-t_1)^2}{ta^2(\epsilon^2-1)^2}r^2+\frac{a^2}{2t}\right]dr$$

$$= \frac{\pi a}{t}\left[\frac{4(t-t_1)^2}{3(\epsilon^2-1)^2}\left(\tau_{s1}\left(\frac{{R_{01}}^3}{a^3}-1\right)+\tau_{s2}\left(\epsilon^3-\frac{{R_{01}}^3}{a^3}\right)\right)\right.$$

$$\left.+a^2\left(\tau_{s1}\left(\frac{R_{01}}{a}-1\right)+\tau_{s2}\left(\epsilon-\frac{R_{01}}{a}\right)\right)\right]$$

$$B_5 = \frac{2\pi a \in}{\in^2 - 1}[\tau_{s1} H + \tau_{s2}(t - t_1 + h - H)]$$

If $\tau_{s1} = \tau_{s2}$ (homogeneous material) then all formulae simplify to known result [1].

To complete the last step we have to substitute all calculated integrals into Lagrange's equation and find minimum of the algebraic expression. Finally we can calculate the resistant force as a function of the depth.

Cases 1, 2, 4 could be considered in the same manner. All integrals can be calculated analytically.

References

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