INSTABILITY OF HIGH-SPEED BOUNDARY LAYER ON POROUS SURFACE

НЕУСТОЙЧИВОСТЬ ВЫСОКОСКОРОСТНОГО ПОГРАНИЧНОГО СЛОЯ НА ПОРИСТОЙ ПОВЕРХНОСТИ

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Abstract: Stability of compressible flat plate boundary layer on porous surface is investigated in the framework of linear stability theory. Stabilizing influence of permeable coating on the second instability mode in hypersonic boundary layer is confirmed. First mode in supersonic Mach 2 boundary layer becomes more unstable under the influence of porosity. However principal possibility of the first mode stabilization is found. It is shown that stabilizing or destabilizing influence of porous coating is determined by phase shift between pressure and normal velocity disturbances at the wall. Existence of a certain optimal phase shift is revealed which leads to a reduction of the boundary layer eigen unstable oscillations growth rates. Combined influence of porosity and surface cooling has also been studied.

1 Introduction. Solution of different technical issues concerning motion of vehicles in fluids or gases requires a control of the boundary layer in such a way that the whole flow acquires qualities desirable for some special purposes. Gas suction out of the boundary layer through a permeable surface is an example of such a control. Experiments show that by means of a suction it is possible to shift boundary layer transition from laminar into turbulent state more downstream. Reynolds number of transition about $\mathrm{Re}_c = 40\cdot 10^6$ can be experimentally obtained in the subsonic flow. Nowadays a great number of theoretical papers exist which explain a stabilizing role of a suction by reduction of the boundary layer thickness and formation of a more stable mean velocity profile.

Thus, both experimental, and the theoretical data indicate a principal possibility of the boundary layer stabilization by means of a suction. More detailed information about flow stabilization at subsonic and also at supersonic speeds can be found in [1, 2] and in a number of other papers. Theoretical papers on suction boundary layer stability usually do not take into consideration some properties of permeable surfaces that can influence the flow stability. Influence of surface properties on stability of a subsonic boundary layer was investigated for the first time in the theoretical paper of S.A.Gaponov [3]. In his subsequent papers he has proposed to use the impedance relation between normal velocity and pressure perturbation at permeable surface, also taking into account compressibility of a gas. Such an impedance relation has successfully been used not only for study subsonic but for low supersonic [4] boundary layers. For a long time [3,4] were the only theoretical papers on the subject, while experimental verification of the theory was not possible at that time. Lack of experiments inversely restrained further development of theoretical modeling. However, a successful experimental investigation of the influence of porous coatings on the stability of Mach $\mathrm{M}=6$ cold hypersonic boundary layer has been performed in [5]. Linear stability of porous surface hypersonic boundary layers in relation to the so called second instability mode was investigated also theoretically in [6] on a basis of given by S.A.Gaponov theoretical impedance relations. So, at present, we are sure that successful experiments are possible also at supersonic speeds where influence of surface permeability on the boundary layer laminar-turbulent transition differs in relation to the case of a hypersonic boundary layer. This distinction is caused by the fact that at supersonic speeds transition is caused primarily by the first (vortical) instability mode while at hypersonic speeds the important role in transition plays the second mode which has an acoustic nature. Investigations of the first mode are more complicated because such waves are oblique (three-dimensional, 3D) while most unstable second instability modes are plane or two-dimensional (2D) waves. Also it is worth to mention that in the range of Mach numbers $3 \le \mathrm{M} \le 5$ a competition of two instability modes takes place, and therefore it is necessary to study both of them.

In the present paper some results of theoretical investigation of linear stability of supersonic Mach $\mathrm{M}=2$ and hypersonic $\mathrm{M}=5.3$ boundary layers are presented.

2 Linear stability analysis. We consider compressible boundary layer on a flat plate assuming a perfect gas with constant Prandtl number $\Pr=0.72$, specific heat ratio $\gamma=1.4$; it was assumed that viscosity μ is a function of temperature only according to the Sutherland relation. In the framework of a linear stability problem the flow-field in a compressible boundary layer can be represented as a combination of the mean flow and a small amplitude disturbance. Basic flow is considered in a self-similar approximation [7]. Equations for disturbance evolution can be obtained by linearization of the equations of motion of viscous compressible heat conducting gas (Navier-Stokes, continuity and energy equations). Solution to the problem can be represented as a combination of harmonic waves:

$$\vec{q}=A(x)\vec{\phi}(y)\exp\left(i\int_{x_0}^{x}\alpha(x')dx'+i\beta z-i\omega t\right),$$

where (x,y,z) are streamwise, normal and spanwise coordinates respectively, wave vector $\vec{k}=(\alpha,\beta)$ is composed of streamwise α and spanwise β wave numbers, $\omega=2\pi f$, f - frequency. For the sought vector $\vec{\phi}=(u,u',v,p,\theta,\theta',w,w')^T$ composed of disturbance velocities (u,v,w), pressure p and temperature θ and their derivatives with respect to y, denoted by primes, one can receive the linear boundary value problem for the system of linear ordinary differential equations:

$$\frac{d\vec{\phi}}{dy}=L(U,T)\vec{\phi}\ , \qquad (1)$$

where L – is the linear Lees-Lin operator of the eighth order. Nonzero elements of L are given in [8] and are functions of the mean streamwise velocity and temperature profiles $(U(y),T(y))$ and of the wave parameters – frequencies and wave numbers. We consider the spatial stability problem, where frequency is regarded as a real value, while streamwise wave number is complex. Streamwise wave number $\alpha=\alpha_{\mathrm{r}}+i\alpha_{\mathrm{i}}$ is determined as an eigenvalue of the boundary value problem, while components of $\vec{\phi}$ are corresponding eigenfunctions. Thus $-\alpha_{\mathrm{i}}>0$ describe unstable disturbances amplifying downstream, while waves with $-\alpha_{\mathrm{i}}\le 0$ are stable and decay with increasing x.

Boundary conditions for (1) at the BL outer edge are usual:

$$|\vec{\phi}|\to 0,\ (y\to\infty), \qquad (2)$$

while boundary conditions for disturbances at permeable surface were developed for the first time in [4-5] and have been applied in the present paper.

We consider a surface of model coated by a porous layer of constant thickness h^* (asterisk denotes dimensional values while variables without asterisk are made nondimensional with the boundary layer length scale). The layer is a flat plate perforated with cylindrical blind holes of constant radius r^* oriented normally to the surface. We assume that pore radius and spacing between adjacent holes s are much smaller in comparison to boundary layer thickness $\delta=\delta(x)$. Under such assumptions boundary conditions for disturbances at the surface can be represented as [4]:

$$u(0)=w(0)=\theta(0)=0,\ v(0)=Kp(0), \qquad (3)$$

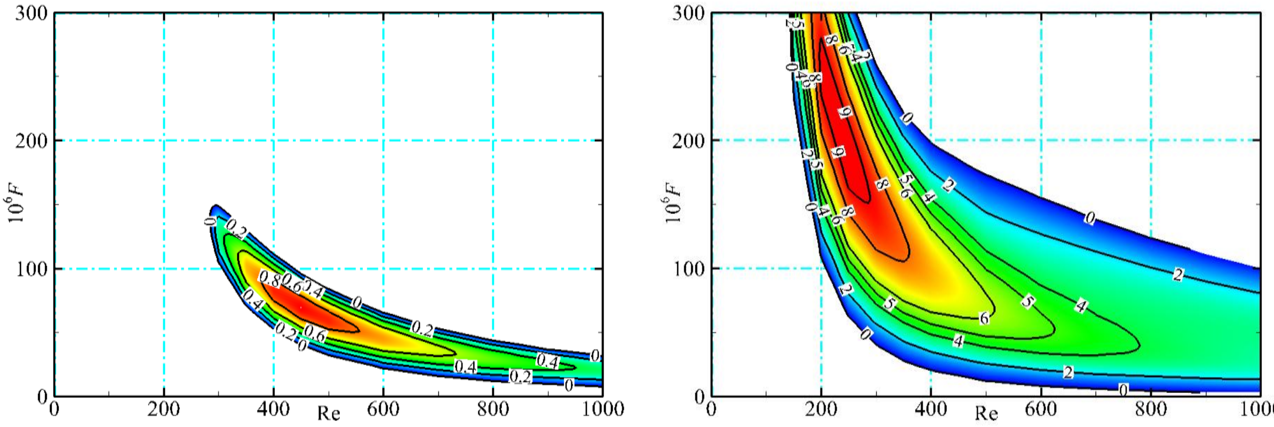
where complex factor K is the acoustic admittance of the porous coating. The magnitude and the phase of K are dependent of porous coating properties, boundary layer properties and wave disturbance parameters. It can be shown [4], that admittance K can be written as

$$K=\frac{n}{Z_0}\tanh(\Lambda h), \qquad (4)$$

where $Z_0 = \sqrt{Z_1 / Y_1}$, $\Lambda = \sqrt{Z_1 Y_1}$, $Y_1 = -i\omega \mathrm{M}^2 \left[\gamma + (\gamma - 1) \frac{J_2\left(k\sqrt{\Pr}\right)}{J_0\left(k\sqrt{\Pr}\right)} \right]$, $Z_1 = \frac{i\omega}{T_w} \frac{J_0(k)}{J_2(k)}$, $k = r\sqrt{\frac{i\omega\rho_w}{\mu_w}\mathrm{Re}}$

.Here M isthe Mach number at the boundary layer outer edge, ρ_w –density, $r = r^* / \delta$ – nondimensional pore radius, $\delta = \sqrt{\mu_e x / U_e \rho_e}$ – Blasius length scale, $\mathrm{Re} = U_e \rho_e \delta / \mu_e$ – Reynolds number, ω – circular frequency, J_0, J_2 – Bessel functions of the corresponding order, n – coefficient of porosity, i.e. part of the surface covered by pores. Subscripts w and e stand for wall and boundary layer outer edge conditions respectively. Stability analysis has been performed by numerical integration of boundary value problem (1–3) by means of method of orthonormalizations [2].

3 Stability of compressible boundary layers on permeable surfaces. Influence of the porous coating on boundary layer stability has been analyzed for supersonic Mach $\mathrm{M}_e = 2$ boundary layer. Figs.1a, b present comparison of the stability diagrams for boundary layer on solid impermeable wall (Fig.1a) and for boundary layer on porous coating with porosity $n = 0.5$ and pore radii $r = 0.5$ (Fig.1b). Contour plots of nondimensionalized spatial amplification rates $-\alpha_i / \mathrm{Re} \cdot 10^6$ in the plane reduced frequency $F = \omega\mu_e / \rho_e U_e^2$ – Reynolds number for two-dimensional (2D, $\chi = \mathrm{arctg}\left(\beta/\alpha_r\right) = 0^\circ$) waves are shown. The region of instability is filled with color. Computations have been performed for deep pores $\left(\tanh(\Lambda h) \to -1\right)$. It is seen that introduction of porous coating leads to considerable enlargement of unstable region and drastic destabilization of the boundary layer. Critical Reynolds number is reduced from $\mathrm{Re}_{cr} \approx 270$ on the solid wall to $\mathrm{Re}_{cr} \approx 140$ on porous wall while the range of unstable frequencies is expanded especially to the region of higher frequencies, from $0 < F \cdot 10^6 < 150$ to $0 < F \cdot 10^6 < 500$ respectively. Maximal amplification rate on porous surface $-\alpha_i$ becomes almost an order of magnitude higher in comparison with nonpermeable surface.



(a) (b)

Fig.1: Contour plot of nondimensional spatial growth rates $-\alpha_i / \mathrm{Re} \cdot 10^6$ in the plane (F, Re) at $\mathrm{M}_e = 2$, $\chi = 0^\circ$: (a) $r = 0$ and (b) $n = 0.5$, $r = 0.5$, $\Lambda h \to \infty$.

Additional computations have been performed to study dependency of the instability wave amplification rates from porous coating thickness h, porosity n and pore sizes r. It was shown that enlargement of the porous coating thickness leads to the increase of amplification rates but this process is not monotonous: there is a certain value of h where $-\alpha_i$ reaches its maximum. Further increase of h will cause a certain reduction of $-\alpha_i$ and then limiting case of deep pores will be achieved $\left(\tanh(\Lambda h) \to -1\right)$. Thickness h at which the limit is reached is dependent of the pore radius and becomes larger for larger pores. For example, for pores with $r^* = 10$ μm the thickness of the porous layer $h \approx 0.5$ mm can be considered as large at $\mathrm{Re} = 600$, $F = 50 \cdot 10^{-6}$.

Investigation of the growth rates of 3D instability waves shows that introduction of porous coating with increasing pore radius leads to monotonous growth of amplification rates for all 3D waves with different orientation angles χ. However this destabilizing influence of porosity is maximal for 2D waves ($\chi = 0$) while with increasing χ the difference in the growth rate with the case of impermeable surface ($r = 0$) become smaller. So, generally, application of the porous coating at Mach $\mathrm{M}_e = 2$ destabilizes boundary layer and accelerates the process of laminar-turbulent transition.

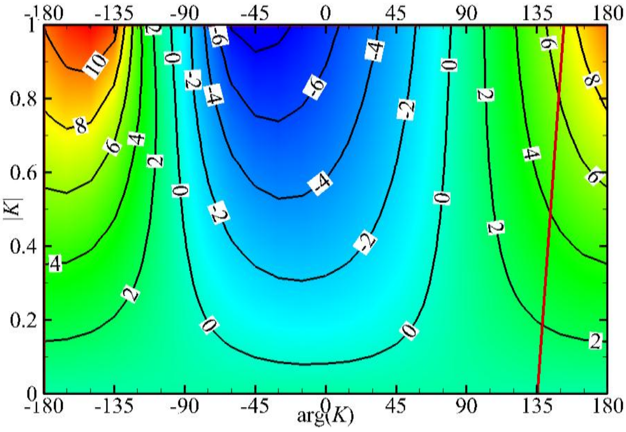


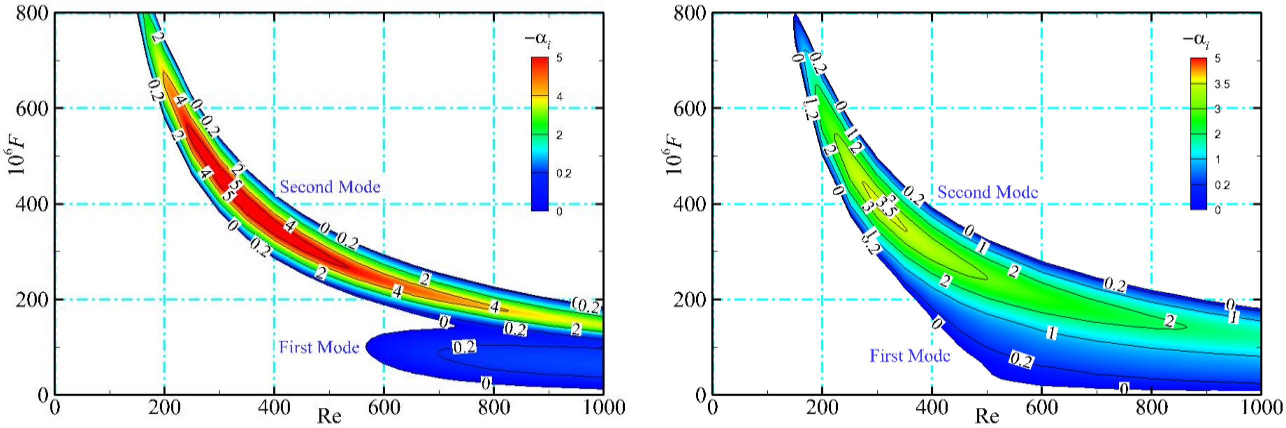
Fig.2: Contour plot of nondimensional spatial growth rates $-\alpha_i / \mathrm{Re} \cdot 10^6$ in the plane $(|K|, \arg K)$ at $\mathrm{M}_e = 2$, $\mathrm{Re} = 600$, $F = 50 \cdot 10^{-6}$, $\chi = 0^\circ$.

Up to now we have considered the influence on boundary layer stability by the porous surface in the form of perforated plate. However now we briefly discuss general case of surface coating which ensures boundary conditions (3) but it is not limited to specific particular case of perforated plate. For a qualitative discussion an arbitrary value, which is not determined by (4), will be given to the admittance K.

Fig.2 shows an example of the contour plot of spatial amplification rates on the plane absolute value of the wall admittance $|K|$ – phase of admittance $\arg K$. Computations have been performed for the wave with $F = 50 \cdot 10^{-6}$, $\chi = 0^\circ$, $\mathrm{Re} = 600$. Fig.2 demonstrates a principal possibility of flow stabilization using special coating. Indeed, it is seen that in fact the character of influence of porous coating is determined by the phase shift between normal velocity and pressure disturbances at the wall (3) which is determined by the argument of the admittance $\arg K$ and depending of these phase shift such an influence can be destabilizing $(90^\circ < |\arg K| < 180^\circ)$ or stabilizing $(-90^\circ < \arg K < 90^\circ)$. Magnitude, the strength of this influence that means change in the growth rate in comparison to the solid surface $|K| = 0$ increases with increasing $|K|$ which can be made by enlargement of the pore radius, or porosity n, or both of them. Real perforated porosity, considered in this paper until now, gives a phase shift $135^\circ < \arg K < 150^\circ$ (shown by the red solid line in the right part of Fig.2) depending on r, but this is located in the region of destabilization. However if it would be possible to fabricate a coating which is able to support a phase shift $-90^\circ < \arg K < 60^\circ$, with optimal $\arg K \approx -30^\circ$ for wave of that frequency, than it would lead to reduction of the disturbance growth rates and, consequently, to the boundary layer stabilization.

Similar computations have been performed for low hypersonic Mach $\mathrm{M}_e = 5.3$ insulated flat plate boundary layer. Stability diagrams for solid and porous walls are shown at Figs.3a, b. At this Mach number the two unstable regions corresponding to the first and second instability modes are present. Comparison of these two diagrams shows that the instability domain stays approximately the same on porous surface. Porous coating attenuates the second mode instability: maximal spatial amplification rate reduces from $-\alpha_i / \mathrm{Re} \cdot 10^6 \approx 6$ to 3.5. Simultaneously instability region of the first mode merges completely with the second mode while corresponding growth rates become larger.

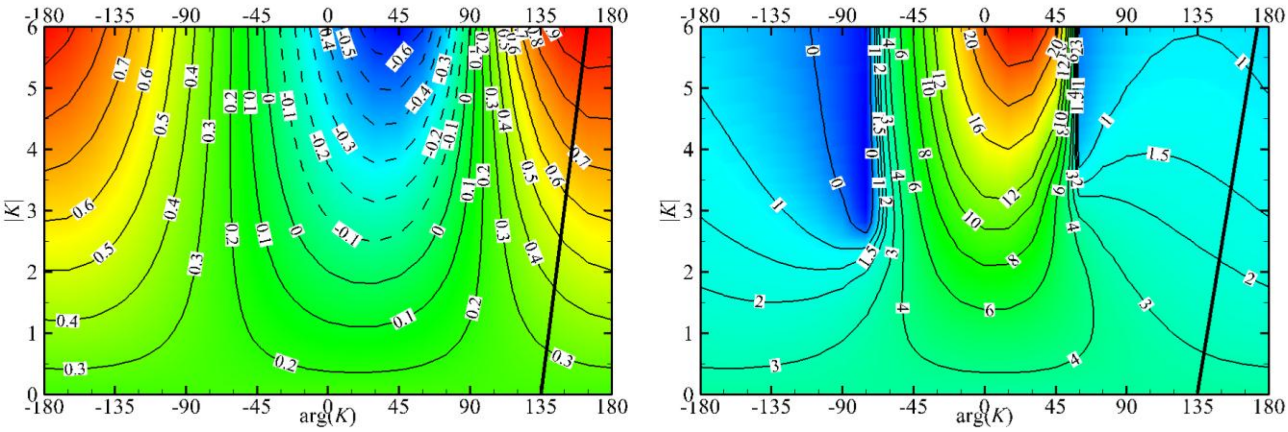
So, the second instability mode which has an acoustic nature and plays dominant role in laminar-turbulent transition in hypersonic boundary layers can be stabilized by the introduction of porous coating of the model surface.



(a) (b)

Fig.3: Contour plot of nondimensional spatial growth rates $-\alpha_i / \mathrm{Re}{\cdot}10^6$ in the plane (F, Re) at $\mathrm{M}_e = 5.3$, $\chi = 0^\circ$: (a) impermeable surface, $r = 0$ and (b)porous coating, $n = 0.5$, $r = 1.0$, $\Lambda h \to \infty$.

Figs.4a, b show contour plot of the growth rates on the plane $|K| - \arg K$ for waves of first and second modes respectively. One can see that for a "realistic" porous coating with increasing $|K|$ along solid lines from $\arg K \approx 135^\circ$ at $|K| = 0$ to $\arg K \to 180^\circ$ at $|K| \to \infty$ porosity gives destabilization of the first mode and stabilization of the second mode.



(a) (b)

Fig.4: Contour plot of nondimensional spatial growth rates $-\alpha_i / \mathrm{Re}{\cdot}10^6$ in the plane $(|K|, \arg K)$ at $\mathrm{M}_e = 5.3$, $\mathrm{Re} = 1040$, $n = 0.5$, $r = 1.0$, $\chi = 0^\circ$: (a)first mode, $F = 58 \cdot 10^{-6}$ and (b)second mode, $F = 140 \cdot 10^{-6}$.

Phase shift responsible for boundary layer stabilization or destabilization is essentially dependent of the internal structure, organization of permeable coating. One example of a structure with permeable surface has been investigated in [9], when flow stabilization was achieved at low subsonic velocities.

4. Combined influence of porosity and surface cooling on stability. Fig.5 shows linear growth rates of disturbances at various values of porosity and three different values of reduced wall temperature for solid impermeable (n=0) and porous coated (n=0.5) walls.

Fig.6 shows linear amplification rates of disturbances at M=5.35. Peak of the growth rate at high value of frequency $F > 150 \cdot 10^{-6}$ correspond to the second instability mode, while another smaller peak (in the low frequency region $F < 120 \cdot 10^{-6}$) correlates with the first (vorticity) mode. One can see that linear growth rates of the second mode are much larger than corresponding values of $-\alpha_i$ of the first mode on impermeable surface. On the porous surface amplification rates of both modes become of the same order of magnitude.

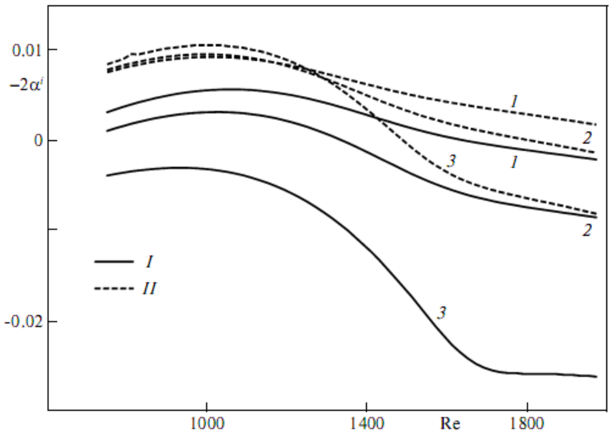


Fig.5: Linear growth rates-α_i of 3D vorticity (first) mode with reduced frequency F=0.2·10^{-4}versus Reynolds number Re: 1-3 - T_W =1.687, 1.4, 1.0 on impermeable (*I*) and porous (*II* - *n*=0.5) surface.

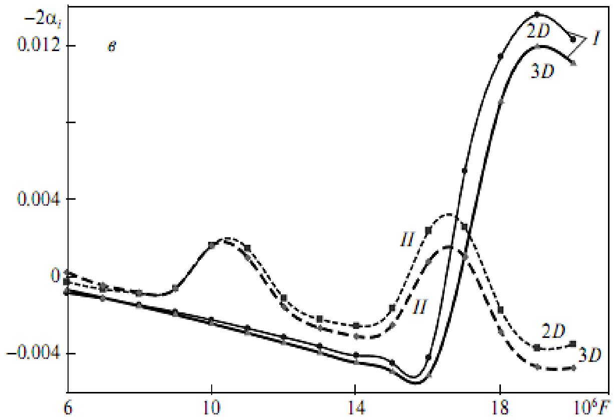


Fig.6:Linear growth rates-α_i of two- (*2D*) and three-dimensional(*3D*) modes versus reduced frequency *F* at Re=1000, T_W=1.587, on impermeable (*I*) and porous (*II* - *n*=0.5) surfaces.

Summarizing, the performed investigation has revealed that wall cooling leads to a stabilization of vorticity perturbations and destabilization of acoustic oscillations in the boundary layer on impermeable surface. Influence of the porous coating on boundary layer stability is just the opposite in comparison to the wall cooling. Porosity amplifies vortical (first mode) instability and attenuates acoustic (second mode) instability.

5 Conclusion. Stability of compressible flat plate boundary layer on porous perforated surface has been studied in the framework of linear stability theory. Performed computations are in a good quantitative agreement with earlier calculations of other authors and confirm stabilizing influence of porous coating on second instability mode. It is shown that first instability mode which is dominating the boundary layer transition process at $\mathrm{M} = 2$ is destabilized on permeable surface independently of all influencing parameters such as pore radius and depth, porosity coefficient, Reynolds number for instability waves of different frequency and orientation.

It is shown that stabilizing or destabilizing influence of the permeable coating depends on the phase shift between pressure and normal velocity perturbations in the instability wave propagating in the boundary layer. Optimal value of such phase shift will ensure suppression of the instability under various flow conditions. Thereby a theoretical principal possibility of supersonic (M=2) boundary layer stabilization on permeable coating is shown.

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