Solvability of a generalized Buckley-Leverett model

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We propose a new mathematical modeling of the Buckley- Leverett system, which describes the two-phase flows in porous media. We prove the solvability of the initial-boundary value problem for a deduced model

$$\partial_t u + \operatorname{div}\big(\mathbf{v}\ g(u)\big) = 0, \tag{1}$$

$$\tau\,\partial_t \mathbf{v} - \nu\Delta\mathbf{v} + h(u)\mathbf{v} = -\nabla p, \qquad \operatorname{div}(\mathbf{v}) = 0, \tag{2}$$

where $u = u(t,\mathbf{x})$ and $\mathbf{v} = \mathbf{v}(t,\mathbf{x})$ are the saturation and the total velocity of the two-phase flow. The parabolic/elliptic type equations (2) are a generalized Darcy Law (Darcy-Brinkman's law when $\tau \neq 0$ / Darcy-Forchheimer's law when $\tau = 0$).

In order to show the solvability result, we consider an approximated parabolic-elliptic system. Since the approximated solutions do not have ANY type compactness property, the limit transition is justified by the kinetic method [1]-[3]. The main issue is to study a linear (kinetic) transport equation, instead of the nonlinear original system.

References

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