DETERMINISTIC LIMIT OF MARKOV MEAN FIELD GAMES

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We will consider the limit of the values of the Markov mean field games where the state space is a countable set $\mathcal{S}$, the dynamics of each player is given by a continuous time Markov chain with a Kolmogorov matrix

$$Q(t, \mu(t), u(t)) = (Q_{x,y}(t, \mu(t), u(t)))_{x,y \in \mathcal{S}}$$

while the payoff functional is equal to

$$\mathbb{E}\left[g(X(T), \mu(T)) + \int_0^T h(t, X(t), \mu(t), u(t)) dt \right].$$

Here $X(t)$ and $\mu(t)$ stand for the state of the player and the distribution of all players respectively.

We discuss the deterministic limit of examined Markov mean field games. The dynamics of the limiting deterministic mean field game is

$$\frac{d}{dt} x(t) = f(t, x(t), m(t), u(t)),$$

where $x(t) \in \mathbb{R}^d$ and $m(t)$ are a state of a player and distribution of all players in the limiting game respectively. The payoff of each agent is given by

$$g(x(T), m(T)) + \int_0^T h(t, x(t), m(t), u(t)) dt.$$

The main result of the talk is a sufficient condition on a sequence of the Markov mean field games that guarantees the convergence (up to subsequence) of the solutions of the Markov mean field games to a solution of the limiting deterministic mean field game. Notice that whether the solution of the deterministic game is unique this result gives an approximation of the solution of the deterministic mean field game by solutions of boundary value problems for ODEs those provide solutions of the Markov mean field games.

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