

Al-Farabi Kazakh National University, Kazakhstan  
Research Institute of Mathematics and Mechanics,  
Kazakhstan

Institute of Computational Mathematics and Mathematical  
Geophysics SB RAS, Russia

Novosibirsk State University, Russia

Shanghai University of Finance and Economics, China

Tianjin University of Finance and Economics, China

Chinese Mathematical Society

hold an

International Conference

**INVERSE PROBLEMS IN  
FINANCE, Economics AND LIFE  
SCIENCES**

Almaty, Kazakhstan, December 26-28, 2017.

## **Program Committee**

**Chair: Galimkair Mutanov, Rector of Al-Farabi Kazakh National University,  
Almaty, Kazakhstan**

Sergey Kabanikhin, Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia Jin Cheng, Fudan University, Shanghai, China Shuhua Zhang, Tianjin University of Finance and Economics, China

Alemdar Hasanoglu (Hasanov) (Izmir University, Izmir, Turkey) Alexander Shananin (Moscow Physical-Technical Institute, Moscow, Russia) Alexandar Ilin (Scientific Center of Anti-Infective Drugs, Almaty, Kazakhstan) Almatbek Kydyrbekuly (Institute of Mathematics and Mechanics, Almaty, Kazakhstan) Abit Asanov (Manas University, Bishkek, Kyrgyzstan) Abdygani Satybaev (Osh University, Osh, Kyrgyzstan) Baojun Bian (Tongji University, Shanghai, China) Bolatbek Rysbayuly (International Information Technology University, Almaty, Kazakhstan) Chenglong Xu (Tongji University, Shanghai, China) Enver Atamanov (Manas University, Bishkek, Kyrgyzstan) Galitden Bakanov (International Kazakh-Turkish University, Turkestan, Kazakhstan) Hongtao Yang (University of Nevada, Las Vegas, USA) Igor Marchuk (Novosibirsk State University, Novosibirsk, Russia) Jaksybek Kulekeyev (Kazakh Institute of Oil and Gas, Astana, Kazakhstan) Jingtang Ma (Southwestern University of Finance and Economics, Chengdu, China) Kali Abdiev (Turan University, Almaty, Kazakhstan) Kazizat Iskakov (L.N.Gumilyov Eurasian National University, Astana, Kazakhstan) Maktagali Bektemesov (Al-Farabi Kazakh National University, Almaty, Kazakhstan) Maksat Kalimoldaev (Institute of Information and Computational Technologies, Almaty, Kazakhstan) Makhmud Sadybekov (Institute of Mathematics and Mathematical modeling, Almaty, Kazakhstan) Maxim Shishlenin (Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia) Mikhail Fedoruk (Novosibirsk State University, Novosibirsk, Russia) Olga Krivorotko (Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia) Tynysbek Kalmenov (Institute of Mathematics and Mathematical modeling, Almaty, Kazakhstan) Talgat Kenjebaev (Kazakh British Technical University, Astana, Kazakhstan) Saltanbek Mukhambetjanov (Scientific Center, Atyrau, Kazakhstan) Yongzeng Lai (Wilfrid Laurier University, Ontario, Canada) Zuoliang Xu (Renmin University of China, Beijing, China)

## **Organizing Committee**

**Chair – Maktagali Bektemesov**

Co-Chair: Dauren Jakebaev, Olga Krivorotko.

Scientific Secretary: A. Alimova, D. Yermolenko.

V. Latyshenko, E. Kondakova, D. Kluychinskiy, N. Novikov, T. Imankul, D. Nurseitov, A. Nurseitov, L. Temirbekova, L. Dairbaeva, S. Kasenov, B. Sholpanbaev, Zh. Bektemessov, A. Turarbek.

# Content

<b>Altayeva A.B.</b> Study of Smart City Platform Based on OneM2M standards . . . . .	<b>7</b>
<b>Asanov A.</b> A CLASS OF SYSTEMS OF LINEAR FREDHOLM INTEGRAL EQUATIONS OF THE THIRD KIND WITH MULTIPOINT SINGULARITIES IN THE AXIS . . . . .	<b>8</b>
<b>Sholpanbaev B.B, Dairbaeva L.M, Askarova Z.B</b> THE PROBLEM OF CONTINUATION OF THE ELECTROMAGNETIC FIELD TO A GIVEN DEPTH . . . . .	<b>9</b>
<b>Bektemessov A.T., Kuvatbayeva A.A.</b> USING OF MODEL CHECKING FOR COMBINING TASKS . . . . .	<b>10</b>
<b>Kabanikhin S.I., Bektemessov M. A., Shishlenin M. A., Yang XS., Bektemessov Z. M.</b> PARAMETER ESTIMATION IN ECONOMIC MATHEMATICAL MODELS USING DIFFERENTIAL EVOLUTION ALGORITHM . . . . .	<b>11</b>
<b>Gorbunov V.K., Lvov A.G.</b> THE PRODUCTION FUNCTION CONSTRUCTION WITH THE EFFECTIVE PRODUCTION FUNDS ESTIMATION . . . . .	<b>12</b>
<b>Kabanikhin S. I., Novikov N. S., Shishlenin M. A.</b> DIRECT LINEAR SEISMIC DATA PROCESSING . . . . .	<b>13</b>
<b>Kabanikhin S. I., Shishlenin M. A.</b> CONTINUATION PROBLEMS IN GEOPHYSICS . . . . .	<b>14</b>
<b>Shishlenin M.A., Kasenov S.E., Askerbekova Zh.A.</b> ALGORITHM FOR SOLVING THE INVERSE PROBLEM FOR THE HELMHOLTZ EQUATION . . . . .	<b>15</b>
<b>Kenzhebaeva M.O.</b> THE RESTORATION OF THE DENSITY OF A HOMOGENEOUS ANOMALY FROM THE MEASUREMENT OF THE GRAVITATIONAL POTENTIAL AND ITS DERIVATIVE ON THE EARTH'S SURFACE . . . . .	<b>16</b>

<b>Kirillov E.V.</b> DIRECT SPECTRAL PROBLEM FOR ONE MATHEMATICAL MODEL OF HYDRODYNAMICS . . . . .	17
<b>Klyuchinsky D. V., Shishlenin M. A.</b> NUMERICAL METHODS FOR THE SOLUTION OF THE CONTINUATION PROBLEM OF A PARABOLIC EQUATION . . . . .	18
<b>Klyuchinskiy D.V., Godunov S.K.</b> AN EXPERIMENTAL INVESTIGATION OF DISCONTINUOUS SOLUTIONS OF A NEW FINITE-DIFFERENCE MODEL OF FLUID DYNAMICS WITH ENTROPY NONDECREASING . . . . .	19
<b>Kondakova E.A., Krivorotko O.I., Kabanikhin S.I.</b> INVERSE PROBLEMS IN FINANCIAL ECONOMICS: OVERVIEW AND ALGORITHMS . . . . .	20
<b>Kozhabekova A.S.</b> THE MODEL OF APPLICATIONS PROCESSING FOR OPTIMAL DELIVERY OF GOODS TO CUSTOMERS IN E-COMMERCE SYSTEM . . . . .	21
<b>Kultaev T.Ch.</b> OPTIMIZATION OF DISTRICTS PURCHASE OF RAW MATERIALS AND THE VOLUME OF PROCESSED PRODUCTS . . . . .	22
<b>Latyshenko V.A., Krivorotko O.I., Kabanikhin S.I.</b> Identifiability of parameters of mathematical models in the field of biology . . . . .	23
<b>Leonov A.S.</b> OPTIMAL AND EXTRA-OPTIMAL METHODS FOR SOLVING ILL-POSED OPTIMIZATION PROBLEMS . . . . .	24
<b>Mamatkasymova A.T.</b> NUMERICAL SOLUTION OF THE DIRECT AND INVERSE PROBLEM OF THE MAXWELL EQUATION WITH INSTANTANEOUS AND LACE SOURCES . . . . .	25
<b>Matanova K.B.</b> ON INVERSE PROBLEM FOR A PSEUDOPARABOLIC THIRD ORDER INTEGRO- DIFFERENTIAL EQUATION WITH VARIABLE COEFFICIENTS . . . . .	26
<b>Mukhambetzhano S.T., Abdiakhmetova Z.M., Shazhdekeeva N.K.</b> ON THE DIRECT AND INVERSE PROBLEM OF THE THEORY OF FILTRATION ON SPECIFICATION OF TECHNOLOGICAL INDICATORS . . . . .	27
<b>Novikov N.S.</b> NUMERICAL SIMULATION OF TWO-DIMENSIONAL ACOUSTIC TOMOGRAPHY	28
<b>Prikhodko A. Yu., Shishlenin M. A.</b> INVERSE PROBLEMS IN THE SOFTWARE DEVELOPMENT OF THE DIGITAL SMART OIL AND GAS FIELD . . . . .	29

<b>Prikhodko A. Yu., Shishlenin M. A.</b> INVERSE PROBLEMS IN MATHEMATICAL MODELS OF CATALYTIC REACTORS	30
<b>Rysbaiuly B., Satybaldina A.N.</b> MATHEMATICAL MODELING OF THE MOLECULAR DIFFUSION MODEL FOR PREDICTING WAX DEPOSITION	31
<b>Satybaev A.D., Alimkanov A.A., Anishenko Yu.V., Kokozova A.Zh.</b> EXISTENCE OF THE SOLUTION OF A TWO-DIMENSIONAL DIRECT PROBLEM OF WAVE PROCESSES WITH INSTANT AND CURRENT SOURCES	32
<b>Serovajsky S., Nurseitov D., Nurseitova A., Azimov A.</b> POSSIBILITIES OF DETERMINATION PARAMETERS OF THE GRAVITATIONAL ANOMALY ACCORDING TO THE RESULTS OF MEASURING THE GRAVITATIONAL FIELD ON THE EARTH SURFACE	33
<b>Shayakhmetov N.</b> FORMULATION OF PROBLEM OF THE OPTIMAL CHOICE OF THE WELL PATTERN FOR THE PRODUCING OF MINERALS USING IN-SITU LEACHING WITH A LIMITED VOLUME OF THE LEACHING SOLUTION	34
<b>Shishlenin M. A.</b> INVERSE PROBLEM FOR THE MATHEMATICAL ECONOMIC MODEL	35
<b>Sigalovsky M.A.</b> OPTIMIZATION METHODS FOR INCORRECT PROBLEMS WITH CONDITIONS ON THE BOUNDARY PARTS IN THE SIMPLE 2D CASE	36
<b>Sultanov M.A., Bakanov G.B.</b> ON THE CONVERGENCE OF SOLUTION OF THE THREE-LAYER PERTURBED DIFFERENCE SCHEME TO SOLUTION OF ILL-POSED CAUCHY PROBLEM	37
<b>Temirbekova L.N.</b> NUMERICAL METHOD FOR SOLVING TWO DIMENSIONAL FREDHOLM INTEGRAL EQUATIONS	38
<b>Turarbek A.T., Begadil Z., Kozhakenov G.</b> SEISMIC RISK ESTIMATION OF ALMATY CITY USING GIS ITRIS	39
<b>Yermolenko D.V., Krivorotko O.I., Kabanikhin S.I.</b> THE SOLUTION OF THE INVERSE PROBLEM OF DETERMINING PARAMETERS FOR THE MATHEMATICAL MODEL OF HIV DYNAMICS	40
<b>Zakirova G.A., Kadchenko S.I.</b> ABOUT ONE INVERSE PROBLEM FOR OCEANOLOGY	41
<b>Zhumat F.S., Duisebekova K.S.</b> THE BLOCKCHAIN HYPE: CURRENT RESEARCH STATE OF TECHNOLOGY	42

**Bidaibekov Y.I., Sholpanbaev B.B., Akimzhan N.Sh.**

The use of digital educational resources in teaching ill-posed system of linear algebraic equations . . . . . **43**

## Study of Smart City Platform Based on OneM2M standards

Altayeva Aizhan Bakatkaliyevna

*Turan university, Almaty*

*aizhan.altayeva@turan-edu.kz*

ICTs are a key factor in smart city initiatives. The integration of ICT in development projects can change the urban landscape and create a number of potential opportunities, the use of which can improve the efficiency of the city's management and functioning of resources and ensure the sustainability of its existence [1]. The development of information and communication technologies (ICT) allows us to formulate new approaches for searching for key factors that determine the success of realizing smart city models and projects. "Smart cities" can be defined as systems that integrate in the framework of a single city space such areas of activity as smart economy; smart mobility; smart environment; smart people; intelligent life; smart management. The future model "smart city" refers to self-actualizing systems, where the rights of access to a huge amount of information in real time belong to both the city's leadership and citizens.

ICT has the main component of the Smart City. The two lower levels here (sensors and data) are the Internet platform of Things (IoT). In other words, the Internet of Things, as a component, is responsible for collecting data in the Smart City.

The priority business areas are: telematics and intelligent transport, medicine, utilities, industrial automation, smart homes, etc. Initially, oneM2M should prepare, approve and maintain the following technical specifications and technical reports: Use cases and requirements for them, taking into account the capabilities of the service layer; high-level architecture for the service layer, taking into account the requirements of independent access to services from end to end; protocols / API / standard objects based on this architecture (open interfaces and protocols); security and privacy aspects; availability and opening of applications; testing rules in accordance with specifications; collection of data for billing and statistical purposes;

The multi-level model oneM2M, provides the provision of M2M services. This model includes three layers: the Application Entity (AE) layer, the Common Service Entity (CSE) layer, and the Network Service Entity (NSE) layer. These functions include device discovery, device and application management, data management and repository, authentication and security functions. The NSE network layer is traditional communication services: launching devices, managing them, including mobile services such as location determination.

### REFERENCES

1. *Altayeva A.B.* [1] The United Nations Economic Commission for Europe, UNECE, the United for Smart Sustainable Cities Initiative (U4SSC) (appeal February 11, 2017).
2. *Kupriyanovsky V.P.* etc. Digital economy = data models + large data + architecture + applications. // International Journal of Open Information Technologies. - 2016. - Vol. 4. - No. 5. - pp. 1-13.

**A CLASS OF SYSTEMS OF LINEAR FREDHOLM INTEGRAL EQUATIONS OF THE THIRD KIND WITH MULTIPOINT SINGULARITIES IN THE AXIS**

Asanov A.

*Kyrgyz-Turkish University Manas, Bishkek**avyt.asanov@mail.ru*

Asanov R.A.

*Kyrgyz State Technical University, Bishkek**ruhidin\_asanov@yahoo.com*

Consider the system of linear integral equations of the third kind

$$p_i(x)u_i(x) = \sum_{j=1}^n \int_{-\infty}^{\infty} k_{ij}(x,y)u_j(y)dy + f_i(x), x \in G = (-\infty, \infty), \quad (1)$$

where  $i = 1, 2, \dots, n$ ,  $p_i(x)$  and  $f_i(x)$  are given continuous functions on  $G$ ,  $k_{ij}(x, y)$  are given continuous functions in  $G \times G$ ,  $u_i(x)$  are the sought functions on  $G$ ,  $i, j = 1, 2, \dots, n$ ; and  $\lambda$  is a real parameter. There exists  $t \in \{1, 2, \dots, n\}$  such that, for all  $i = t, t + 1, \dots, n$  and  $l = 1, 2, \dots, m(i)$ ,  $p_i(x_{il}) = 0$ , where  $x_{il} \in G$  and for all  $i = 1, 2, \dots, t - 1$ ,  $p_i(x) = 1$  for all  $x \in G$ .

Various issues concerning the theory of integral equations were studied in [1–3]. Here, a new approach is proposed for the study of systems (1). Following this approach, we prove that the solution of system (1) in the space  $L_{2,n}(G)$  is equivalent to the solution of systems of linear integral equations of the second kind with the some integral conditions. Here  $L_{2,n}(G)$  denote the space of all  $n$ -dimensional vector-functions with elements of  $L_2(G)$ .

**Keywords:** system, third kind, multipoint singularities, axis.

**2010 Mathematics Subject Classification:** 45B05, 45A05, 45G10.

## REFERENCES

1. *Laurent'ev M.M., Romanov V.G., Shishatskii S.R.* Ill-Posed Problems of Mathematical Physics, 1986, Amer. Math. Soc., Providence.
2. *Imanaliev M. I., Asanov A., Asanov R.A.* A class of Systems of Linear Fredholm Integral Equations of the Third kind // Doklady Mathematics, 2011, vol.83, no.2,, Pp. 227-231.
3. *Imanaliev M. I., Asanov A., Asanov R.A.* Solutions to Systems of Linear Fredholm Integral Equations of the Third kind with Multipoint Singularities, 2017, vol.95, no.3,, Pp. 1-5.



**THE PROBLEM OF CONTINUATION OF THE ELECTROMAGNETIC FIELD  
TO A GIVEN DEPTH**

Sholpanbaev B.B, Dairbaeva L.M, Askarova Z.B

*Al-Farabi Kazakh National University, Almaty*

*bahtygerey@mail.ru, lazat.dairbayeva@gmail.com, zulfia\_94.94@list.ru*

We consider the problem of continuation of the electromagnetic field from a part of the boundary  $x = 0$  in the region  $\Omega \times (0, T)$  where  $\Omega = \{(x, y) : x \in (0, h), y \in (0, L)\}$ , [1]-[2].

$$u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}u_t = u_{xx} + u_{yy}, \quad (x, y) \in \Omega, \quad t \in (0, T), \quad (1)$$

$$u|_{t=0} = 0, \quad u_t|_{t=0} = 0, \quad (x, y) \in \Omega, \quad (2)$$

$$u_x(0, y, t) = g(y, t), \quad y \in (0, L), \quad t \in (0, T), \quad (3)$$

$$u(0, y, t) = f(y, t), \quad y \in (0, L), \quad t \in (0, T). \quad (4)$$

$\varepsilon$  - dielectric permittivity of the medium,  $\mu$  - magnetic permeability of medium,  $\sigma$  - conduction,  $u(x, y, t)$  - horizontal component of the electric field strength vector of the electromagnetic field. The problem (1) - (3) is incorrect.

To solve the initial problem, we consider an auxiliary is correct problem for equation (1) with conditions (2), (3) and additional boundary conditions

$$u(h, y, t) = q(y, t), \quad y \in (0, L_y), \quad t \in (0, T), \quad (5)$$

$$u(x, 0, t) = u(x, L_y, t) = 0, \quad x \in (0, h), \quad t \in (0, T). \quad (6)$$

The problem (1) - (3), (4), (5) will be called a direct problem, where, by the given  $g(y, t)$  and  $q(y, t)$  it is required to determine  $u(x, y, t)$ .

The inverse problem is considered, which consists in determining the function  $q(y, t)$  from equation (1), conditions (2), (3), (4), (5) and additional information (3).

It is shown that the solution of the ill-posed problem (1) - (3) reduces to solving the inverse problem (1) - (5). To solve the inverse problem (1) - (5), that is, to find the function  $q(y, t)$ , a computational algorithm was developed based on the Landweber optimization method.

REFERENCES

1. 1 *Kabanikhin S.I., Iskakov K.T., Sholpanbaev B.B.* Analysis of the measurements inhomogeneities (archaeological objects) using Georadar // The 8th international congress of the ISAAC, 2011, Moscow, P. 292-293.
2. 2 *Kabanikhin S.I., Shishlenin M.A., Nurseitov D.B., Sholpanbaev B.B.* The continuation problem for the equation of electrodynamics // Vestnik of KazNU. A series of mathematics, mechanics, computer science, 2013, No. 4(79), P. 66-84.

## USING OF MODEL CHECKING FOR COMBINING TASKS

*Turan University, Almaty*  
*a.bektemessov@turan-edu.kz*

One of the most promising and widely used approaches to solving the problem of debugging automation and verifying the correctness of programs is Model Checking. For a given analyzed program, its abstract formal model is constructed. The property or requirement to be tested is expressed in the formal method in the form of a logical formula:  $M \models \phi$ , mean that a some Boolean formula  $\phi$  satisfies the models M which verifies the program is to verify the feasibility of the formalized specification requirement on the abstract program model [1, 2]. In this work, we try to solve combinatorial problems using Model Checking.

Model checking is not only for detecting errors in the program, you can also find answers for some logical tasks. For example, you can find solutions to the Rubik's Cubic. Finding a solution is quite difficult. To sort out all possible variants in the Promela language:

```
inline rotate_side_by_f(rot)
...
inline perm(x,y,x1,y1) /*permission for rotations*/
active proctype Rubik()
{
check();
do
/*Infinite cycling to find all states*/
od}
```

Approval specification:

"In the future, all lines are sorted by color."

In the logic of LTL:

$F(colors - done \wedge low - path)$  or  $\langle \rangle (colors - done \wedge low - path)$  where, *colors - done* - predicate of truth.

It is very convenient to solve all possible variants for solving problems using the Model Checking method. Finding a solution to a problem using temporal logic with the opposite statement is one of the true way by the logic of CTL. That allows you to find ways to solve the Rubik's Cube.

### REFERENCES

1. *Karpov Yu. G.* MODEL CHECKING Verification of parallel and distributed software systems // St. Petersburg, 2010.
2. *Edmund M.C., Thomas A.H., Helmut V.* Handbook of Model Checking // Springer International Publishing, 2016, P-1212 .

**PARAMETER ESTIMATION IN ECONOMIC MATHEMATICAL MODELS  
USING DIFFERENTIAL EVOLUTION ALGORITHM**

Kabanikhin S.I.<sup>1,2,3</sup>, Bektemessov M. A.<sup>4</sup>,

Shishlenin M. A.<sup>1,2,3</sup>, Yang X.S.<sup>5</sup>, Bektemessov Z. M.<sup>6</sup>

<sup>1</sup>*Institute of Computational Mathematics and Mathematical Geophysics SB RAS, 630090, Novosibirsk*

<sup>2</sup>*Sobolev Institute of Mathematics SB RAS, 630090, Novosibirsk*

<sup>3</sup>*Novosibirsk State University, 630090, Novosibirsk*

<sup>4</sup>*Abai Kazakh National Pedagogical University, 050010, Almaty*

<sup>5</sup>*Middlesex University, NW4 4BT, London*

<sup>6</sup>*al-Farabi Kazakh National University, 050040, Almaty*

*kabanikhin@scc.ru, maktagali@mail.ru, mshishlenin@ngs.ru, x.yang@mdx.ac.uk,*

*jolaman252@gmail.com*

Nowadays economics plays a huge role in human’s life, because it is in every part of our daily routine: food, clothes, home, entertainment and many others. Peoples’ economic independence is the one of the most valuable things that is why we go to work and earn money. Also no one wants to waste hard earned money, so we try to optimize it by mathematical tools and create some economical forecast. It is not only the personal issue, but also of the countries and government, where interesting roles play the population growth and capital accumulation.

$$\dot{k}(t) = s_k k^\alpha(t) - (n + g + \delta)k(t) \quad (1)$$

There was completed process of restoring the coefficients, by solving the inverse problem, using the algorithm of differential evolution [2], for Robert Solow’s mathematical model (1) of neoclassical economic growth, based on the Cobb-Douglas production function, taking into account labor, capital and exogenous neutral technical progress [1]. Also, the model proposed by Mankiw-Romer-Weil (2) with the addition of human capital is considered, where the number of variables and unknown coefficients is already increasing [3].

$$\begin{cases} \dot{k}(t) = s_k k(t)^\alpha h(t)^\beta - s_k k(t) \\ \dot{h}(t) = s_h k(t)^\alpha h(t)^\beta - s_h h(t) \end{cases} \quad (2)$$

REFERENCES

1. *Solow R. M.* Contribution to the Theory of Economic Growth // The Quarterly Journal of Economics. 1956. V. 70, Issue 1. P. 65–94.
2. *Yang X.S.* Nature-Inspired Optimization Algorithms. Elsevier, 2014
3. *Mankiw G. N., Romer D., Weil D. N.* A Contribution to the Empirics of Economic Growth // The Quarterly Journal of Economics. 1992 V. 107, Issue 2. P. 407–437.
4. *Storn R., Price K.* Differential Evolution — A Simple and Efficient Adaptive Scheme for Global Optimization over Continuous Spaces. Technical Report TR-95-012, ICSI, 1995.

## THE PRODUCTION FUNCTION CONSTRUCTION WITH THE EFFECTIVE PRODUCTION FUNDS ESTIMATION

Gorbunov V.K., Lvov A.G.  
*Ulyanovsk state university, Ulyanovsk*  
*vkgorbunov@mail.ru*

Effective production funds (EPFs) of a country, region, and sectors of economy are a part of balance (inventory) funds participating in creation of goods and services in actual market conditions. In the production functions (PFs) representing complex production objects, the effective funds, but not balance ones, have to be used. It is possible to construct such PFs and to estimate EPFs simultaneously in the case when production statistics contains production investment data instead of or together with capital ones. This possibility was realized in [1] where the model of production investments capitalization was constructed.

The model consists of the standard PF model having EPFs as the main factor (another are labor and/or energy quantities) and the funds' dynamic equation. The latter is defined by investments as well as a depreciation rate and, optionally, a lag of investment capitalization. The identification of this complex model gives simultaneously a PF's parameters and value of EPFs estimations. Specifics of the problem are the absence of the error estimations of production statistics which are initial data for the identification, and it is commonplace for social sciences. A regularization of the identification problem demands usage of additional expert information and non-trivial optimization technique. In view of natural analytical constraints on PFs (monotonicity and quasiconcavity) this problem can be considered as the isogeometrical approximation of a table-given function which is the production output depended of EPFs, defined implicitly, and another factors.

The work [2] develops the approach of [1]. As an additional means for overcoming of computational complexities, the transform to the index form of PFs was used. In the new paper [3] the method of EPFs estimation is extended on small business where the index of production funds is absent in view of its ill-definiteness (rooms and facilities are used for living needs as well as for production), and a new regularization condition on the initial and final values of the EPFs is introduced.

The research is supported by the Russian Foundation of Basic Research. Project No. 16-06-00372

### REFERENCES

1. *Gorbunov V.K., Lvov A.G.* The construction of production functions via investment data // *Economica i matematicheskie metody*, 2012, Iss. 2, P. 95-107.
2. *Gorbunov V.K., Krylov V.P.* Effective regional production funds and their estimation by the method of production function // *Economy of Region*, 2015, No. 3, P. 334-347.
3. *Gorbunov V.K., Lvov A.G.* Effective production funds and production functions of regional small business // *Economy of Region*, 2018 (Submitted, Russian).

**DIRECT LINEAR SEISMIC DATA PROCESSING**Kabanikhin S. I.<sup>1,2,3</sup>, Novikov N. S.<sup>1,3</sup>, Shishlenin M. A.<sup>1,2,3</sup><sup>1</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*<sup>2</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia*<sup>3</sup>*Novosibirsk State University, Novosibirsk, Russia**kabanikhin@sccc.ru, novikov-1989@yandex.ru, mshishlenin@ngs.ru*

Currently, due to the area systems of observations, it is possible to create a fundamentally new method of solving three-dimensional inverse problems, which are used: three-dimensional analogues of the equations of Gelfand-Levitan-Krein [1, 5], parallel computing on high performance clusters, methods Monte-Carlo [3, 4], fast algorithms for the inversion of block-Toeplitz matrices of large dimensions [2].

The main problem of the investigations of three-dimensional elastic media is the large size of the area in which high-precision calculations are produced. Even for a relatively small area of 2 km × 2 km × 2 km the solution of the direct problem of seismic prospecting is a very difficult. Let us note that the most of the modern methods for solving inverse problems are based on iterative procedures and even the number of operations required for carrying out multiple iterations, may lead to uncontrolled errors.

The work was supported by Russian Foundation for Basic Research (projects NNo. 16-29-15120 and 15-01-09230).

## REFERENCES

1. *Kabanikhin S.I., Satybaev A.D., Shishlenin M.A.* Direct Methods of Solving Multidimensional Inverse Hyperbolic Problems. VSP, The Netherlands, 2004.
2. *Kabanikhin S.I., Novikov N.S., Oseledets I.V., Shishlenin M.A.* Fast Toeplitz linear system inversion for solving two-dimensional acoustic inverse problem // *J. of Inverse and Ill-Posed Problems*, 2015. Vol. 23, No. 6, pp. 687–700.
3. *Kabanikhin S.I., Sabelfeld K.K., Novikov N.S., Shishlenin M.A.* Numerical solution of an inverse problem of coefficient recovering for a wave equation by a stochastic projection methods // *Monte Carlo Methods and Applications*. 2015. Vol. 21, No. 3. P. 189–203.
4. *Kabanikhin S.I., Sabelfeld K.K., Novikov N.S., Shishlenin M.A.* Numerical solution of the multidimensional Gelfand–Levitan equation // *Journal of Inverse and Ill-Posed Problems*, 2015. Vol. 23, No. 5, pp. 439–450.
5. *Kabanikhin S.I., Shishlenin M.A.* Numerical algorithm for two-dimensional inverse acoustic problem based on Gelfand-Levitan-Krein equation // *Journal of Inverse and Ill-Posed Problems*. 2011. Vol. 18, No. 9. P. 979–996.

**CONTINUATION PROBLEMS IN GEOPHYSICS**Kabanikhin S. I.<sup>1,2,3</sup>, Shishlenin M. A.<sup>1,2,3</sup><sup>1</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*<sup>2</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia*<sup>3</sup>*Novosibirsk State University, Novosibirsk, Russia**kabanikhin@scc.ru, mshishlenin@ngs.ru*

We consider continuation problems in Geophysics and numerical methods for their solution. The continuation problems of physical fields with the data on the part of the boundary [1, 2, 3] are ill-posed.

Continuation problems are formulated in the form of operator equation  $Aq = f$ , for which the minimization of the objective functional and the method of singular value decomposition [2, 3] are applied.

We study the properties of the operator  $A$  and the algorithm of minimization of functional  $J(q) = \|Aq - f\|^2$  by the conjugate gradient method. In series of numerical experiments are shown that it allows us to recover the boundary conditions on the inaccessible part of the boundary, as well as to obtain information about inhomogeneities (the number, location, approximate volume) located in the region of inaccessibility.

The work was supported by Russian Foundation for Basic Research (projects NNo. 17-51-540004, 16-29-15120, 16-01-00755 and 15-01-09230).

## REFERENCES

1. *S.I. Kabanikhin, D. Nurseitov, M.A. Shishlenin, B.B. Sholpanbaev. Inverse Problems for the Ground Penetrating Radar // Journal of Inverse and Ill-Posed Problems. 2013. Vol. 21. No. 6, pp. 885–892.*
2. *S. I. Kabanikhin, Y. S. Gasimov, D. B. Nurseitov, M. A. Shishlenin, B. B. Sholpanbaev and S. Kasenov. Regularization of the continuation problem for elliptic equations // Journal of Inverse and Ill-Posed Problems. 2013. Vol. 21. No. 6. Pp. 871–884.*
3. *Kabanikhin S.I., Shishlenin M.A., Nurseitov D.B., Nursetova A.T., Kasenov S.E. Comparative analysis of methods for regularizing an initial boundary value problem for the Helmholtz equation // Journal of Applied Mathematics. 2014. Vol. 2014, 7 pages.*

## ALGORITHM FOR SOLVING THE INVERSE PROBLEM FOR THE HELMHOLTZ EQUATION

Shishlenin M.A., Kasenov S.E., Askerbekova Zh.A.

*ICMMG SB RAS, Al-Farabi KazNU, Almaty*

*mshishlenin@ngs.ru, syrym.kassenov@kaznu.kz, ask-janar@mail.ru*

We consider the initial-boundary value problem for the Helmholtz equation, which is ill-posed. We reduce the solution of the operator equation  $Aq = f$  to the problem of minimizing the functional  $J(q) = \langle Aq - f, Aq - f \rangle$ . We construct an algorithm for solving the inverse problem [1].

### Algorithm for solving the inverse problem

1. We choose the initial approximation  $q^0 = (q_1^0, q_2^0)$ ;
2. Let us assume that  $q_n$  is known, then we solve the direct problem numerically

$$\begin{aligned} u_{xx} + u_{yy} - \left( \frac{\rho_x}{\rho} u_x + \frac{\rho_y}{\rho} u_y \right) + \left( \frac{\omega}{c} \right)^2 u &= 0, & (x, y) \in \Omega, \\ u(0, y) = h_1(y), \quad u(1, y) &= q_1^n(y), & y \in [0, 1], \\ u(x, 0) = h_2(x), \quad u(x, 1) &= q_2^n(x), & x \in [0, 1]. \end{aligned}$$

3. We calculate the value of the functional

$$J(q_{n+1}) = \int_0^1 [u_x(0, y; q_1^{n+1}, q_2^{n+1}) - f_1(y)]^2 dy + \int_0^1 [u_y(x, 0; q_1^{n+1}, q_2^{n+1}) - f_2(x)]^2 dx;$$

4. If the value of the functional is not sufficiently small, then go to next step;
5. We solve the conjugate problem

$$\begin{aligned} \psi_{xx} + \psi_{yy} + \left( \frac{\rho_x}{\rho} \psi \right)_x + \left( \frac{\rho_y}{\rho} \psi \right)_y + \left( \frac{\omega}{c} \right)^2 \psi &= 0, & (x, y) \in \Omega, \\ \psi(0, y) = 2(u_x(0, y; q_1, q_2) - f_1(y)), \psi(1, y) &= 0, & y \in [0, 1], \\ \psi(x, 0) = 2(u_y(x, 0; q_1, q_2) - f_2(x)), \psi(x, 1) &= 0, & x \in [0, 1]. \end{aligned}$$

6. We calculate the gradient of the functional  $J'(q^n) = (-\psi_x(1, y), -\psi_y(x, 1))$ ;
7. We calculate the following approximation  $q^{n+1} = q^n - \alpha J'(q^n)$ ;

### REFERENCES

1. Kabanikhin S.I., Shishlenin M.A., Nurseitov D. B., Nurseitova A.T., Kasenov S.E. Comparative Analysis of Methods for Regularizing an Initial Boundary Value Problem for the Helmholtz Equation // Journal of Applied Mathematics Volume 2014, <http://dx.doi.org/10.1155/2014/786326>

**THE RESTORATION OF THE DENSITY OF A HOMOGENEOUS ANOMALY  
FROM THE MEASUREMENT OF THE GRAVITATIONAL POTENTIAL AND  
ITS DERIVATIVE ON THE EARTH'S SURFACE.**

Kenzhebaeva M.O.

*Al-Farabi Kazakh National University, Almaty  
merey-mex-2017@mail.ru*

The problem of studying the deep structure of the earth's crust is one of the strategic directions of geophysical research, providing the development of Earth sciences. In this gravity prospecting is one of the main methods of studying the structure of the earth's crust. Gravimetric or gravitational reconnaissance (abbreviated gravity prospecting) is a geophysical method for studying the structure of the lithosphere, prospecting and prospecting for minerals, based on the study of the Earth's gravitational field. This leads to the need to solve some inverse problems in which the results of measuring the gravitational potential need to restore the parameters of the system under consideration.

We consider a rectangular area with horizontal and vertical coordinates. In this region there is some gravitational anomaly. The location of the anomaly and its shape are considered known. We assume the homogeneity of the material of both the anomaly itself and the space outside it. In this case, the density of the material outside the anomaly is assumed to be known, and the density of the anomaly itself is to be determined in the process of solving the corresponding inverse problem. Having established this density, we can judge what this anomaly represents: it is some concrete material or, possibly, a void.

The mathematical model describing the process is reduced to the Poisson equation with respect to the gravitational field, the right-hand side of which contains the required density. Boundary conditions: on the outer surface, the potential of the gravitational field and its derivative along the normal are measured. To determine the boundary conditions on the rest of the boundary, we assume that the dimensions of the anomaly are small enough and it is removed from the boundary of the region under consideration. Then we can assume that the value of the potential on the specified part of the boundary will be the same as in the absence of an anomaly. As a result, we obtain the Dirichlet problem for the Poisson equation with an additional boundary condition on the outer boundary. Based on this additional information, it is also intended to determine the density of the anomaly.

The problem is reduced to minimizing the functional expressing the mean square deviation of the derivative of the potential along the vertical coordinate from the measurement results. We propose to solve the problem by a gradient method. For this, the derivative of the functional is defined. Using the gradient method, we obtain an iterative process, at each step of which the initial boundary value problem and the conjugate system are solved, and the transition to the new iteration is performed on the basis of the standard technique.



**DIRECT SPECTRAL PROBLEM FOR ONE MATHEMATICAL MODEL OF  
HYDRODYNAMICS**

Kirillov E.V

*South Ural State University, Chelyabinsk*

*kirillovev@susu.ru*

In this paper we consider a direct spectral problem for a mathematical model of hydrodynamics, that includes an operator with bounded perturbation in the case when it's spectrum is multiple. A similar problem was considered earlier for an operator with single spectrum [1]. The main method of research is modified method of regularized traces. We introduce the relative resolvent of the operator. A spectral problem of the form  $(M + P)u = Lu$  is obtained. In this case, the operator  $L$  is such that the relative resolvent of the operator is a nuclear operator. As a result of applying the resolvent method to the relative spectrum of the perturbed operator, we obtain relative eigenvalues of the perturbed operator with non-nuclear resolvent.

Let  $P$  be operator of multiplication by a function  $p \in C^2(\Pi)$ . Consider operator  $T + P$ . Denote  $\{\nu_n\}_{n=1}^{\infty} = \sigma^L(T + P)$  — where  $\nu_n$  are numbered in order of nonincrease in their real parts, taking into account their multiplicity. The following problem is solved: find the relative eigenvalues  $T + P$ . The result is

$$\sum_{q=1}^{\eta_n} \nu_n^q = \eta_n \mu_n + \sum_{q=1}^{\eta_n} (L^{-1} P \varphi_n, \varphi_n) + \alpha_n, \quad (1)$$

where

$$\alpha_n = \sum_{k=2}^{\infty} \frac{(-1)^k}{2\pi i} \int_{\gamma_n} \mu \operatorname{Sp}[R_0(\mu)P]^k L R_0(\mu) d\mu.$$

Dzektser equation

$$(a^2 - \Delta)u_t = \alpha \Delta u - \beta \Delta^2 u + f$$

is modeling the evolution of the free surface of a filtering fluid. Parameters  $\alpha, \beta > 0$ ,  $a^2 \in \mathbb{R}$  characterize properties of the environment, free member  $f = f(x)$  corresponds to source (drains) of fluid.

Define operators  $T, L : U \rightarrow F$

$$T = \alpha \Delta - \beta \Delta^2, \quad \Delta = \sum_{i=1}^n \frac{\partial^2}{dx_i^2}, \quad L = a^2 - \Delta, \quad (2)$$

Then the relative eigenvalues of  $T + P$  can be founded by formula (2).

The work has been supported by Act 211 Government of the Russian Federation, contract No 02.A03.21.0011 REFERENCES

1. *Kirillov E.V.* The Spectral Identity for The Operator With Non-nuclear Resolvent // *Jornal of Computational and Engineering Mathematics*, 2017, Chelyabinsk, pp. 69-75.

Klyuchinsky D. V.<sup>1,3</sup>, Shishlenin M. A.<sup>1,2,3</sup>

<sup>1</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*

<sup>2</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia*

<sup>3</sup>*Novosibirsk State University, Novosibirsk, Russia*  
*dmitriy\_klyuchinskiy@mail.ru, mshishlenin@ngs.ru*

The continuation problem (the Cauchy problem) for the parabolic equation with a data on the part of the boundary is investigated [1]. This problem is ill-posed and a solution depends on the applied numerical method. That is why the comparison of different numerical methods is required. We compare the finite-difference scheme inversion, the singular value decomposition and gradient type method. The influence of a noisy data on the solution is presented.

The work was supported by Russian Foundation for Basic Research (projects NNo. 17-51-540004, 16-29-15120 and 16-01-00755).

#### REFERENCES

1. *Belonosov, A., Shishlenin, M.* Regularization methods of the continuation problem for the parabolic equation (2017) Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 10187 LNCS, pp. 220–226.

**AN EXPERIMENTAL INVESTIGATION OF DISCONTINUOUS SOLUTIONS  
OF A NEW FINITE-DIFFERENCE MODEL OF FLUID DYNAMICS WITH  
ENTROPY NONDECREASING**

Klyuchinskiy D.V., Godunov S.K.

*Novosibirsk State University, Sobolev Institute of Mathematics of SB RAS  
dmitriy\_klyuchinskiy@mail.ru, godunov@math.nsc.ru*

In this work we introduce a new linearized finite-difference model of fluid dynamics with condition of entropy nondecreasing on discontinuous solutions (shock waves). This model is modern logical development of the canonical work [1], based on the linearized analogue of the Riemann problem solution from the book [2]. The model has been built by using equations of fluid dynamics 1-3

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(p + \rho u^2)}{\partial x} = 0 \quad (2)$$

$$\frac{\partial H}{\partial t} + \frac{\partial(u(H + p))}{\partial x} = 0, \quad (3)$$

with classical equation of state  $E = \frac{p}{(\gamma-1)\rho}$ , that represent the conservation laws of mass, impulse and energy. Here in the model  $H = \rho \frac{u^2}{2} + \frac{p}{\gamma-1}$  is enthalpy function. The pressure of a gas is determined with formula  $p = \sigma(S)\rho^\gamma$ , where  $S$  is entropy variable. The velocity of speed is found as  $c = \sqrt{\gamma p/\rho}$ . We do not include the conservation law of entropy in our system because the law of entropy nondecreasing is hold automatically due to inner structure of finite-difference model.

Using the designed model the structure of shock waves has been investigated. It was shown the dependence of a width of shock waves and their formation time on the changing of the Courant number. Also we explored three different configurations of the Riemann problem: shock wave and rarefaction wave, two rarefaction waves and two shock waves. On these tests we described the finite-difference features and effects, that can influence on the solution. It was noticed, that these negative effects come to zero while reducing the grid step. Again, the numerical fulfillment of the law of entropy nondecreasing has been shown here.

The work has been supported by the Ministry of Education and Science of the Russian Federation (4.1.3 The Joint Laboratories of NSU-NSC SB RAS).

REFERENCES

1. *Godunov S.K.* A Finite Difference Method for the Computation of Discontinuous Solutions of the Equations of Fluid Dynamics. // Mat. Sb., 47:357–393, 1959.
2. *Godunov S.K., Zabrodin A.V., Ivanov M.Ya., Krayko A.N. and Prokopov G.P.* Numerical Solution of the Multidimensional Gas Dynamics Problems, 1976, pages 130-132, Moscow: Nauka.

## INVERSE PROBLEMS IN FINANCIAL ECONOMICS: OVERVIEW AND ALGORITHMS

Kondakova E. A., Krivorotko O.I., Kabanikhin S.I.  
*Novosibirsk State University, Novosibirsk*  
*ekondak95@mail.ru, olga.krivorotko@sscc.ru, kabanikhin@sscc.ru*

In this paper we present a review of mathematical models in the financial economy. Mathematical models in the financial economy are divided into deterministic ones, which are described by systems of ordinary differential equations (ODE) [1] or parabolic equations, and stochastic ones are described by systems of stochastic differential equations [2]. We also consider statements of inverse problems [3] that define a function on the right-hand side (control function). For example, a control function that is responsible for the optimal rate of transfer from the bond holdings to the stock and the rate of opposite transfers [2] or an optimal investment strategy [4]. Algorithms for solving such problems based on the theory of optimal control are presented.

*The work has been supported by the grant no. 1746/GF4 of the Ministry of Education and Science of Republic of Kazakhstan, Ministry of Education and Science of Russian Federation and by the Scholarship of the President of RF no. MK-1214.2017.1.*

### REFERENCES

1. *S. I. Chernyshov, V. S. Ponomarenko, A. V. Voronin.* The Problem of Modelling of Economic Dynamics in Differential Form // Cornell University Library, 2008, P. 36.
2. *H. Mete Soner.* Stochastic Optimal Control in Finance // 2004, Oxford, P. 62.
3. *S. I. Kabanikhin.* Inverse and Ill-Posed problems // Berlin: de Gruyter, 2011.
4. *Thomas A. Weber.* Optimal Control Theory with Applications in Economics // Massachusetts Institute of Technology, 2011.

## THE MODEL OF APPLICATIONS PROCESSING FOR OPTIMAL DELIVERY OF GOODS TO CUSTOMERS IN E-COMMERCE SYSTEM

Kozhabekova A.S.

*International Information Technologies University, Almaty*

*akosyakozhabekova@gmail.com*

E-commerce consists of a variety of processes, starting with the processes of the stage of procurement and delivery of goods to the warehouse, ending with the processes of delivering goods to customers. Therefore, for the correct and effective improvement and development of the e-commerce system, the formalization of all e-commerce processes, the formulation of tasks for the optimal performance of these processes, and an effective algorithm for their solution, taking into account the relationship, are primarily required based on systemic positions.

In order to ensure that all these steps are worked optimally and smoothly, it is necessary to consider all the processes of e-commerce in a comprehensive manner, covering all the specialized processes that are part of the overall business process.

In this paper, based on the above mentioned approaches, the processes of primary processing of applications are examined: reception, registration, verification of the reliability of submitted applications, formalization of content.

Thus, the stage of preparation of applications for formalized processing of applications is considered. We will ask the question posed by introducing the model of primary application processing. First, we will distinguish the role of every parameter involved in this process. We need to formalize the processes of admission and content of applications in order to have better vision of the process. At the current time, a project-application for the purchase of goods by the buyer (applicant) is submitted (or received). Submission of the application is made by filling in the application form of the e-store application. If in the text, when filling, there are spelling errors and inaccuracies, they are eliminated along with the applicant in the interactive model.

Before accepting and registering, project proposals are tested for reliability of the applicant. One way to assess the reliability of the buyer is to analyze the buyer's behavior from historical data, as previously ordered by this buyer and how he behaved. The reliability of the buyer or applicant can also be determined from historical data from other resources or the Internet by searching for data about the correctness and reliability of this applicant. This process is similar to the banking process of checking the reliability of a future borrower in the clearance process.

So, the process of accepting applications consists of two stages. At the first stage, the buyer enters the project-application. The correctness of filling the project-application is checked. If the applicant's reliability is confirmed, the project application is accepted by the system as an application and it is registered, after which the content of the application is formalized and the identification code is received.

An algorithm for the entire process of accepting an application and its formalization is proposed.

**OPTIMIZATION OF DISTRICTS PURCHASE OF RAW MATERIALS AND  
THE VOLUME OF PROCESSED PRODUCTS**

Kultaev T.Ch.

*Osh State University, Kyrgyzstan*

*kt\_1958.2005@mail.ru*

Formulation of the problem. Suppose that the region has the enterprise  $A_0$ , where raw materials are processed and received from it  $r$  – type of product ( $r = 1, 2$ ). Manufactured products are sold at wholesale prices. Enterprise  $A_0$  buys raw materials from the region  $n$  areas at a specific price for each area  $B_j, j = 1, 2, \dots, n$ . It is required to determine the best plan for the purchase of raw material enterprise  $A_0$  from areas  $B_j, j = 1, 2, \dots, n$ , and the volume of products delivering enterprise  $A_0$  maximum net income.

We introduce the following notation for the formalization of the problem:  $j$  – index of area the purchase of raw material of enterprise  $A_0, j \in J; J$  – index set raw material procurement areas in the region where  $J = 1, 2, \dots, n; r$  – index type of product,  $r \in R; R$  – index set of products derived from the processing company, where  $R = 1, 2, \dots, p$ .

$$L(x, y) = \sum_{r \in R} S_0^r y_0^r - \sum_{j \in J} \bar{C}_{0j} x_{0j} \quad (1)$$

under conditions

$$0 \leq x_{0j} \leq \alpha_{0j}, \quad i \in J, \quad (2)$$

$$\sum_{i \in J} a_j^r x_{0j} = y_0^r, \quad r \in R, \quad (3)$$

$$\sum_{j \in J} \bar{C}_{0j} x_{0j} = D, \quad (4)$$

$$y_0^r \geq 0, \quad r \in R, \quad (5)$$

where  $x = |x_{0j}|_{1, |J|}, \quad y = |y_0^r|_{1, |R|}$ .

REFERENCES

1. *Asankulova M., Jusupbaev A.* Optimization of production and distribution of raw materials among users depending upon the period. Comprehensive science problems and education. 2016, No. 4 (46), pp. 7–12.
2. *Asankulova M., Jusupbaev A., Jusupbaeva G.A.* Determination of the maximum income of the enterprise with a limited amount of finance. Actual trends of research of the XXI century: Theory and practice, 2017, Vol. 3, No. 7, part 1 (18-1), pp. 101-105.

**Identifiability of parameters of mathematical models in the field of biology**

Latyshenko V.A., Krivorotko O.I., Kabanikhin S.I.

*Novosibirsk State University, Novosibirsk**Institute of Computational Mathematics and Mathematical Geophysics of SB RAS, Novosibirsk**Latushenko\_varia@mail.ru, krivorotko.olya@mail.ru, kabanikhin@scc.ru*

Systems of ordinary differential equations (ODEs) are among the most widely used tool for describing the dynamic processes of our life. These systems have many applications in the field of physics, information technology, life sciences, etc. ODEs have received much attention over the last two decades in the field of biology (immunology, epidemiology, pharmacokinetics etc). Coefficients of such mathematical models describe individual characteristics of patient and population and should be identify for construction of individual treatment plan and the best forecasting of epidemic.

Before determining unknown parameters of models (inverse problem [1]), we should understand: whether is there a solution, how many parameters can we determine from the available data, how many measurements (additional information about solution of ODEs in fixed times) need to be taken to determine the required set of parameters? These questions are answered by analysis of the identifiability [2]. The identifiability is the ability to uniquely determine the values of parameters with sufficient data volume. Analysis of the identifiability represents a study for a deep understanding of the model.

In this work we analyse several methods of identifiability analysis (practical and sensitivity-based) and apply its to different mathematical models of biology (model of dynamic HIV-infection, model of spread of co-infection HIV and tuberculosis and others).

*The work has been supported by the grant no. 1746/GF4 of the Ministry of Education and Science of Republic of Kazakhstan, Ministry of Education and Science of Russian Federation and by the Scholarship of the President of RF no. MK-1214.2017.1.*

## REFERENCES

1. *Kabanikhin S.I.* Definitions and examples of inverse and ill-posed problems // J. Inverse Ill-Posed Probl., 2008, V.16, pp. 317–357.
2. *Miao H., Xia X., Perelson A.S., We H.* On identifiability of nonlinear ODE models and applications in viral dynamics // SIAM Rev. Soc. Ind. Appl. Math., 2011, V.53(1), pp. 3–39.

**OPTIMAL AND EXTRA-OPTIMAL METHODS FOR SOLVING ILL-POSED  
OPTIMIZATION PROBLEMS**

Leonov A.S.

*National Nuclear Research University "MEPHI Moscow  
asleonov@mephi.ru*

The concept of the quality of approximate solutions to ill-posed optimization (or extremal) problems is introduced and a posteriori quality estimates for various solution methods are studied. Examples of quality functionals are given, which can be used in solving practical extremal problems. These functionals include standard norms in Banach spaces, as well as non-standard quality estimators like BV-variation and mathematical entropy. New concepts of optimal, optimal in order and extra-optimal quality of the method for solving the extremal problem are determined. The theory of stable methods for solving extremal problems with optimal in order and extra-optimal quality is developed, in which, in particular, the property of consistency for quality estimators is studied. Examples of regularizing algorithms with extra-optimal quality of solutions for extremal problems are given. The presented results are the development of the work [1].

The work has been supported by the Russian Foundation for Basic Research (grants no. 16-01-00450-a, 17-01-00159-a, 17-01-53002-NSFC-a)

## REFERENCES

1. *Leonov A.S.* Regularizing algorithms with optimal and extra-optimal quality // Numerical Analysis and Applications, 2016, Vol. 9, No. 4, P. 369–380.



**NUMERICAL SOLUTION OF THE DIRECT AND INVERSE PROBLEM OF  
THE MAXWELL EQUATION WITH INSTANTANEOUS AND LACE  
SOURCES**

Mamatkasymova A.T.  
*Osh Technological University, Kyrgyzstan*  
*mamatkasymova1973@mail.ru*

The propagation of electromagnetic waves in a medium at a certain external current describes Maxwell's system equations [1].

The system of Maxwell's equations, when the parameters  $\varepsilon, \mu, \sigma$  do not depend on variables  $x_1, x_2$ , can be reduced to a one-dimensional problem [2].

$$\varepsilon(x_3) \frac{\partial^2 E_3(x_3, t)}{\partial t^2} = \frac{1}{\mu(x_3)} \cdot \left[ \frac{\partial^2 E_3}{\partial x_3^2} \right] - \frac{\mu'_{x_3}(x_3)}{\mu^2(x_3)} \frac{\partial E_3}{\partial x_3} - \sigma(x_3) \frac{\partial^2 E_3}{\partial t^2}, \quad (x_3, t) \in R_+^2, \quad (1)$$

where  $E(x_3, t)$  – electric tension,  $\varepsilon(x_3), \mu(x_3)$  – dielectric and magnetic permeability,  $\sigma(x_3)$  – electrical conductivity of the medium.

We give the initial and boundary conditions in the following form:

$$E_3(x_3, t)|_{t < 0} \equiv 0, \quad \left. \frac{\partial E_3(x_3, t)}{\partial x_3} \right|_{x_3=0} = \frac{1}{\varepsilon(0)} r_0 \delta(t) + \frac{1}{\varepsilon(0)} h_0 \theta(t), \quad (2)$$

where  $\delta(t), \theta(t)$  – the Dirac function, the Heaviside theta function,  $r_0, h_0$  – positive constants.

To formulate the inverse problem, it is necessary to specify a temporary-like additional condition of the following form:

$$E_3(x_3, t)|_{x_3=0} \equiv f(t), \quad t \in [0, T]. \quad (3)$$

A direct problem is to determine the function  $E_3(x_3, t)$  from problem (1) – (2) for known values of the coefficient parameters of the equations  $\varepsilon(x_3), \mu(x_3), \sigma(x_3)$ . The inverse problem consists in restoring one of the coefficients of the equations from problem (1) – (3) with the known two other coefficients of the equations.

In this paper, we construct a numerical finite-difference solution of the direct and inverse problem, show the convergence of the approximate solution to the exact solution, establish the estimate of the convergence of the solution, and prove the stability of the solution.

REFERENCES

1. *Romanov V.G., Kabanikhin S.I.* Inverse problems of geoelectrics. Moscow: Science, 1991.
2. *Mamatkasymova A.T., Satybaev A.D.* The inverse problem of the system of Maxwell's equations with the distributed current in the cable. News of Universities. Bishkek, 2014, No.3., pp. 7–19.

**ON INVERSE PROBLEM FOR A PSEUDOPARABOLIC THIRD ORDER  
INTEGRO-DIFFERENTIAL EQUATION WITH VARIABLE COEFFICIENTS**

Matanova K.B.

*Kyrgyz-Turkish Manas University, Bishkek*

*mkalyskan@gmail.com*

It is considered the inverse problem of restoration of the kernel and the unknown right-hand side of a third-order pseudoparabolic integro-differential equation with variable coefficients. The sufficient conditions of existence and uniqueness solution of the inverse problem are received by using integral equations method [2] and Green function method [1].

by using integral equations method and Green function The sufficient conditions of existence and uniqueness solution of the inverse problem are received Statement of the problem. It is required to find an unknown functions  $u(x, t)$ ,  $K(t)$  and  $\varphi(t)$  satisfying in the domain  $G = \{(x, t) : a \leq x \leq b, 0 \leq t \leq T\}$  the following equation

$$u_t(x, t) = a_0(Au)'_t + a_1(Au) + b_0(x, t) \frac{\partial^2 u(x, t)}{\partial x^2} + b_1(x, t) \frac{\partial u(x, t)}{\partial x} + b_2(x, t)u(x, t) + \int_0^t K(t-s)u(s, x)ds + \varphi(t)f(x, t) + F(x, t) \quad (1)$$

and the initial, boundary and additional conditions

$$u(x, 0) = u_0(x), u(a, t) = 0, u(b, t) = 0, x \in (a, b), t \in [0, T], \quad (2)$$

$$u(x_i, t) = g_i(t), i = 1, 2, t \in [0, T], x_i \in (a, b), x_1 < x_2, \quad (3)$$

where  $0 < T$ ,  $a_0 \neq 0$ ,  $a_1$  are given constants,  $b_0(x, t)$ ,  $b_1(x, t)$ ,  $b_2(x, t)$ ,  $f_i(x, t)$ ,  $F(x, t)$ ,  $u_0(x)$ ,  $g_1(t)$ ,  $g_2(t)$  are given functions,  $A$  is a linear differential operator of the second order with the variable coefficients of the form

$$Au(x, t) = \frac{\partial^2 u}{\partial x^2} + p(x, t) \frac{\partial u}{\partial x} + q(x, t)u.$$

REFERENCES

1. *Collatz, L.* Eigenwertaufgaben mit technischen Anwendungen, Leipzig, 1963.
2. *Krasnov, M.L.* Integral Equations: An Introduction to the Theory Nauka, Moscow, 1975) (in Russian)

**ON THE DIRECT AND INVERSE PROBLEM OF THE THEORY OF  
FILTRATION ON SPECIFICATION OF TECHNOLOGICAL INDICATORS**

Mukhambetzhano S.T.<sup>1</sup>, Abdiakhmetova Z.M.<sup>2</sup>, Shazhdekeeva N.K.<sup>1</sup>

<sup>1</sup> *Atyrau State University named after Kh.Dosmukhamedov*; <sup>2</sup> *al Farabi Kazakh National University, Almaty*

<sup>2</sup> *zukhra.abdiakhmetova@gmail.com*

The work is devoted to the investigation of the problem of pressure refinement in the areas of power supply and unloading and identification of technological indicators in the near-well zone of the formation. Concentration of transfer of individual components can be described by the equation of convective diffusion

$$mS_r \frac{\partial C_r}{\partial t} + \overset{\rho}{v}_r \nabla C_r - D_r \nabla^2 C_r = 0 \quad (1)$$

$D_r$  - coefficient of dispersion, calculated by the formula

$$D_r = D_0 \left[ \frac{1}{F^* m} + 0.5 \frac{\vec{U}_r d_p \sigma}{m D_0} \right]^n; \overset{\rho}{U}_r = - \frac{K_r(x, y)}{\mu_r} \nabla P_r \quad (2)$$

The filtration of a multicomponent mixture is described by a system of equations

$$\operatorname{div} \rho_r h \overset{\rho}{U}_r + m h S_r \frac{\partial \rho_r}{\partial t} + q_r = 0 \quad (3)$$

$$\operatorname{div} \rho_r h C_i \overset{\rho}{U}_r + m h S_r \frac{\partial \rho_r C_i}{\partial t} + q_r C_i = 0; i = 1, n; \rho_r = \rho_0 \frac{\rho_r T_0}{\rho_0 T_z} \quad (4)$$

$$\sum_{i=1}^n C_i = 1 \quad (5)$$

The initial conditions are

$$T = T_0; p_r = p_0(x, y, t); C_i = C_{i0}(x, y, t); i = i, n - 1 \quad (6)$$

The boundary conditions are as follows

$$F(x, y) = 0; f(p_r \frac{\partial P_r}{\partial n}, x, y, t) = 0; C_i = C_{ir}(x, y, t) = 0; i = 1, n - 1 \quad (7)$$

The direct problem of convective diffusion consists in finding functions  $P_r$  and  $C_i$ , satisfying equations (4) - (5), the initial conditions, the boundary conditions. The functions  $q_r(x, y, t)$ ,  $k_r(x, y)$ ,  $m(x, y)$  and  $h(x, y)$  are assumed to be given. The inverse problem for convective diffusion can be in determining the parameters  $k_r$ ,  $m$  and  $h$  satisfying equations (4) - (11) if the data are known  $P_r(x, y, t)$  and  $C_i(x, y, t)$  in a certain part of the filtration area at certain points in time. Numerical experiments with real data were carried out.

## NUMERICAL SIMULATION OF TWO-DIMENSIONAL ACOUSTIC TOMOGRAPHY

Novikov N.S.

*Novosibirsk state university, Institute of computational mathematics and mathematical geophysics, Novosibirsk  
novikov-1989@yandex.ru*

We consider the direct and inverse problem for the hyperbolic system of the two dimensional acoustic wave propagation. These problems are related to ultrasound tomography for the detection of the inclusions in the soft human tissue.

The hyperbolic first-order system of the acoustics allows us to propose realistic model from the physical point of view. To solve the inverse problem we apply the gradient method of minimization of the functional for recovering coefficients of the system. We apply the numerical algorithm for solving the direct problem based on the S.K. Godunov scheme [1]. This approach allows us to find the balance between mathematical modeling of physical process and the effective numerical simulation.

The propagation of the acoustic waves in the 2D medium is described by the following system:

$$\rho u_t + p_x = 0, \quad \rho v_t + p_y = 0, \quad p_t + \rho c^2(u_x + v_y) = f(x, y)r(t), \quad (x, y) \in \Omega, \quad t \in (0, T). \quad (1)$$

Here  $u = u(x, y, t)$ ,  $v = v(x, y, t)$  are components of velocity vector,  $p = p(x, y, t)$  is the acoustic pressure,  $\rho(x, y)$  is the density of the medium,  $c(x, y)$  is the speed of the wave propagation,  $f(x, y)$  is the source location,  $r(t)$  is the Ricker impulse.

Inverse problem is to find  $\rho(x, y)$  and  $c(x, y)$  by known additional information  $p(x_k, y_k, t) = f_k(t)$ , where  $\{(x_k, y_k)\}_{k=1}^K$  are location of the receivers [2].

The formulation of the acoustic equations in the form of conservation make it possible to develop efficient numerical scheme for solving direct and inverse problems. These scheme allows us to simulate also the the radiation pattern of the source and add the absorption coefficient. The numerical scheme enforces conservation laws, which are the main reason to control the solution, and in future this scheme can be efficiently implemented on high performance computers with a multiprocessor architecture.

*The work was supported by RFBR under grants 15-01-09230, 16-01-00755 and 16-29-15120, the Ministry of Education and Science of the Russian Federation and the Ministry of Education and Science of the Republic of Kazakhstan, grant MES 1746/GF 4.*

### REFERENCES

1. I.M. Kulikov, N.S. Novikov, M.A. Shishlenin, *Mathematical modelling of ultrasound wave propagation in two dimensional medium: direct and inverse problem*, Siberian Electronic Mathematical Reports **12** (2015), C.219–C.228.
2. S.I. Kabanikhin, I.M. Kulikov, N.S. Novikov, M.A. Shishlenin, *Acoustical tomography based on the the two dimensional hyperbolic system of acoustic equations*, Journal of Inverse and Ill-Posed Problems (2018) (to appear).

**INVERSE PROBLEMS IN THE SOFTWARE DEVELOPMENT OF THE  
DIGITAL SMART OIL AND GAS FIELD**Prihodko A. Yu.<sup>1</sup>, Shishlenin M. A.<sup>1,2,3</sup><sup>1</sup>*Novosibirsk State University, Novosibirsk, Russia*<sup>2</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*<sup>3</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia**prihodko1997@gmail.com, mshishlenin@ngs.ru*

One of the important problem of well diagnostics is the operational determination of the change in well production and water cut [1]. We propose an algorithm for estimating these parameters by solving the inverse problem. In the inverse problem, it is required to determine the amount of fluid coming from the formation into the well (flow rate) and the ratio of the amount of water to the amount of oil (water cut), from the measured pressure and temperature at a given depth [2].

The importance of solving the direct and inverse problems for a two-phase flow (liquidity and gas) in a well is determined by the fact that currently only about one hundred thousand wells are operated in Russia. The installation of special equipment, which allows for continuous monitoring of well operations, is a very complicated and expensive process. Monitoring can be realized if the sensors of pressure and temperature included in the standard set of submerged pump telemetry are used, and sensors for measuring pressure and temperature are installed at almost every well.

The work was supported by Russian Foundation for Basic Research (projects NNo. 17-51-540004, 16-29-15120 and 16-01-00755).

## REFERENCES

1. *S.I. Kabanikhin, A.N. Cheremisin, M.A. Shishlenin*. The Inverse Problem of Determining Water Cut and Flow in a Vertical Fountain Well // *Siberian Journal of Industrial Mathematics*. 2011. T. 14, No. 3 (47). Pp. 31–36.
2. *A.E. Ryazantsev, S.I. Kabanikhin, M.A. Shishlenin*. Mathematical justification of the use of submersible pump telemetry systems for continuous monitoring of production wells // *Vestnik TsKR Rosnedra*. 2013. Vol. 5. P. 32–36.

**INVERSE PROBLEMS IN MATHEMATICAL MODELS OF CATALYTIC  
REACTORS**Prihodko A. Yu.<sup>1</sup>, Shishlenin M. A.<sup>1,2,3</sup><sup>1</sup>*Novosibirsk State University, Novosibirsk, Russia*<sup>2</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*<sup>3</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia**prihodko1997@gmail.com, mshishlenin@ngs.ru*

The inverse problems in mathematical models of catalytic reactors are considered. The new algorithms for identifying the mechanisms of reactions from the experimental dependences of the concentrations of individual compounds are developed based on the metaheuristic, gradient and Newtonian methods for solving the inverse problem. Comparative analysis of the developed algorithms is presented.

The work was supported by Russian Foundation for Basic Research (projects NNo. 17-51-540004, 16-29-15120 and 16-01-00755).

## MATHEMATICAL MODELING OF THE MOLECULAR DIFFUSION MODEL FOR PREDICTING WAX DEPOSITION

Rysbaiuly B.<sup>1</sup>, Satybaldina A.N.<sup>2</sup>

<sup>1,2</sup>*International Information Technology University, Almaty*

<sup>1</sup>*b.rysbaiuly@mail.ru*, <sup>2</sup>*aigul1191@gmail.com*

Deposition of high molecular weight paraffins on the inner wall of subsea production and transportation pipelines continues to be a critical operational problem faced by the petroleum industry. Molecular diffusion is employed in the vast majority of the deposition models available in the literature, it is still not clear which one is the most relevant mechanism. In the present study a numerical analysis of the wax deposition is performed employing only the molecular diffusion model. The wax deposition is determined considering oil flowing in the laminar regime, through a rectangular channel. [1]

Wax deposition in production and transportation pipelines continues to be a relevant problem for the industry, particularly in offshore operations. The main objective of the research is to help identify the relative importance of the wax deposition mechanisms. [2]

The formulation is two-dimensional and the wax deposition rate is a function of the paraffin concentration gradient. The finite volume method was selected to solve the conservation equations of mass, energy and mass fraction, coupled with a diffusive equation to describe the growth of the wax deposit. The basic goal is to construct a mathematical and computer models of finding parameters of wax deposition rate.

This work is aimed to find industrial parameters of the underground pipeline. It was considered a square pipeline, which consists of oil, paraffin, steel, thermal insulation by glass wool.

### REFERENCES

1. *Leiroz, A. T., Romero, M. I., Nieckele, A.O., Azevedo, L.F.A.* Wax Deposition in Laminar Channel Flow // Proceedings of 18th International Congress of Mechanical Engineering, ABCM, November 6-11, 2005, Ouro Preto, MG, Brazil.
2. *Romero, M. I.* Assessment of Molecular Difusion as a Mechanism for Wax Deposition in Petroleum Pipelines // MSc. Dissertation, Pontificia Universidade Catolica do Rio de Janeiro – PUC-Rio, Rio de Janeiro, Brazil (in portuguese), 2005.

**EXISTENCE OF THE SOLUTION OF A TWO-DIMENSIONAL DIRECT  
PROBLEM OF WAVE PROCESSES WITH INSTANT AND CURRENT  
SOURCES**

Satybaev A.D., Alimkanov A.A., Anishenko Yu.V., Kokozova A.Zh.  
*Osh Technological University, Kyrgyzstan*  
*abdu-satybaev@mail.ru*

The work is devoted to the study of natural complex waves, namely the existence of a solution to the problem of wave processes.

Many wave processes of geophysics, electrodynamics, seismics and others, are characterized by fields described by second-order equations of mathematical physics

$$\begin{aligned} \operatorname{div}(\bar{\tau}(x, y, z)\operatorname{grad}\bar{\vartheta}(x, y, z, t)) - \bar{a}(x, y, z)\bar{\vartheta}(x, y, z, t) - \bar{b}(x, y, z)\frac{\partial\bar{\vartheta}(x, y, z, t)}{\partial t} - \\ - \bar{c}(x, y, z)\frac{\partial^2\bar{\vartheta}(x, y, z, t)}{\partial t^2} = -\bar{f}(x, y, z, t), \end{aligned} \quad (1)$$

where  $(x, y, z) \in R^2 \times R_+$ ,  $t \in R_+$ ,  $\bar{a}, \bar{b}, \bar{c}$  - numerical coefficients are physical parameters,  $\bar{f}(x, y, z, t)$  - the intensity function of the field sources,  $\bar{\tau}(x, y, z)$  - a function describing geophysical media,  $\bar{\vartheta}(x, y, z, t)$  - the field potential.

REFERENCES

1. *Romanov V.G.* Stability of inverse problems. - M.: The scientific world, 2004, P. 304.
2. *Kabanikhin S.I.* Inverse and ill-posed problems. Novosibirsk: Siberian Scientific Publishing House, 2009, P. 457.
3. *Satybayev A.D.* Existence of the solution of the direct problem of the wave equation with a plane boundary // Proceedings of the II International Scientific and Methodological Conference "Mathematical Modeling and Information Technologies in Education and Science Almaty, 2003, pp. 383–389.
4. *Satybayev A.D.* Existence of the solution of the direct problem of acoustics with a flat boundary // Science and New Technologies, No. 1, Bishkek, 2006, pp. 164–172.
5. *Satybaev A.D., Anishenko Yu.V.* The existence of a solution to the direct problem of a geoelectrics with a plane boundary and a cabled source // Bulletin of the KRSU, 2017, Vol. 17, No. 8, pp. 18–22.
6. *Kokozova A.Zh., Satybayev A.D.* The existence of a solution to the two-dimensional direct problem of a telegraph equation with an instantaneous and a cabled source // JOURNAL of the Kyrgyz State Technical University named after I. Razzakov, No. 2 (35), part 1, Bishkek, 2017, pp. 175–180.
7. *Satybaev A.D., Alimkanov A.A.* The existence of a solution to the two-dimensional direct seismic problem with an instantaneous source, Science and New Technologies, No. 7, Bishkek, 2014, pp. 3–6.



**POSSIBILITIES OF DETERMINATION PARAMETERS OF THE  
GRAVITATIONAL ANOMALY ACCORDING TO THE RESULTS OF  
MEASURING THE GRAVITATIONAL FIELD ON THE EARTH SURFACE**

Serovajsky S., Nurseitov D., Nurseitova A., Azimov A.

*Al-Farabi Kazakh National University, Almaty*

*Satbayev Kazakh National Research Technical University, Almaty*

*serovajskys@mail.ru, ndb80@mail.ru, altynna@mail.ru, anvar.aa@mail.ru*

One of the most important areas of exploration and analysis of mineral deposits is the determination and identification of gravity anomalies. This is the analysis of the different deviations of the gravitational potential based on gravimetric methods. These results make it possible to judge the presence of any inhomogeneities in the thickness of the earth's crust in question.

We consider the Poisson equation for the potential of the gravitational field in a certain region. The inverse problem is to determine the density of the ground based on measuring the gravitational potential and its derivative on the surface of the earth. We suppose that the value of the potential at the remaining part of the boundary of the region under consideration is equal to such its value that would be observed in the same region in the absence of a gravitational anomaly.

We analyze the following cases.

1. Determination of the density distribution throughout the region in the absence of any information about the location of the anomaly.
2. Determination of the density distribution with known place and unknown depth of location of the anomaly.
3. Determination of the density distribution with known depth of location of the anomaly and unknown horizontal location.
4. Determination of the density and the depth of location of the homogeneous anomaly.
5. Determination of the depth of location of the homogeneous anomaly with known the density.
6. Determination of the density the homogeneous anomaly with known form and location of the anomaly.

By the performed calculations, we have the following conclusions. We cannot guaranty the satisfactory information about the properties of the anomaly by the measuring of distribution of the gravitational potential and its gradient over the earth's surface in the absence of any information on the shape, size, location and structure of the gravitational anomaly. Nevertheless, we can determine a reliable enough estimate of its horizontal location for this case. However, the presence of any a priori information about the anomaly makes it possible to increase seriously the accuracy of the calculations. The determination of more accurate results in the general case requires the use of additional experimental data.

**FORMULATION OF PROBLEM OF THE OPTIMAL CHOICE OF THE WELL  
PATTERN FOR THE PRODUCING OF MINERALS USING IN-SITU  
LEACHING WITH A LIMITED VOLUME OF THE LEACHING SOLUTION**

Shayakhmetov N.

*Al-Farabi Kazakh National University, Almaty  
shayakhmetovn@gmail.com*

All uranium mines in Kazakhstan are developing by In-situ leaching method, which allows obtaining a low production cost of raw materials at high safety levels for the ecology and health of employees. This method is based on injection of leaching solution to the ore formation and production of productive solution with certain concentration of uranium [1]. However, because of the complexity of the chemical processes and the geological structure of the deposit as well as the low cost of uranium raw materials, geological and technological processes require high accuracy that can be provided by mathematical modeling using modern computer technologies [2]. Summarizing the foregoing, can be concluded that modern methods of optimization are required.

Mathematical modeling of technological processes is based on the equations and laws of hydrodynamics, the kinetics of chemical reactions in which the input parameters are the flow rate or pressure at the wells, concentration of the injected leaching agent and initial concentration of uranium in the formation. Hydrodynamic modeling based on the mass conservation law and Darcy's equation makes it possible to determine stagnant, spreading and dilution zones. Concluding, In-situ leaching process can be described by system of partial differential equations.

The author consider the problem of optimal planning and designing. The optimal planning problem is determined on selecting of debits in the injection wells. The optimal designing includes the selection of the optimal well pattern with the highest amount of produced uranium. At present, linear and hexagonal (cellular) well patterns are widely used. Therefore, the management of production is carried out on the basis of the regulation of debits at the technological assembly nodes and the optimal drilling of wells according to the location of the ore body. Limitations were placed on the volume of injected fluid during the entire production period; the number of wells is limited by the outline of the ore-bearing formation. Main object is to find optimal number of wells in the limited area with a limited amount of injecting leaching solution (in producing period) to take maximal amount of produced uranium.

REFERENCES

1. In Situ Leach Uranium Mining: an Overview of Operations. International Atomic Energy Agency, 2016.
2. *Boytsov A.* World uranium industry: state, development prospects, challenges of time // VIII International research and practical conference "Contemporary issues of uranium mining industry 2017, Astana.

**INVERSE PROBLEM FOR THE MATHEMATICAL ECONOMIC MODEL**Shishlenin M. A.<sup>1,2,3</sup><sup>1</sup>*Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk, Russia*<sup>2</sup>*Sobolev Institute of Mathematics, Novosibirsk, Russia*<sup>3</sup>*Novosibirsk State University, Novosibirsk, Russia**mshishlenin@ngs.ru*

In this work we consider the mathematical problem of the economy, connecting such parameters as GDP, labor, labor productivity, capital, technology and capital adequacy. The inverse problem of determining the correlation functions between considered parameters of the mathematical model is solved based on the analysis of USA data for 40 years.

The work was supported by Russian Foundation for Basic Research (projects NNo. 17-51-540004, 16-29-15120 and 16-01-00755).

## OPTIMIZATION METHODS FOR INCORRECT PROBLEMS WITH CONDITIONS ON THE BOUNDARY PARTS IN THE SIMPLE 2D CASE

Sigalovsky M.A.

*Al-Farabi Kazakh National University, Almaty*

*mark.sgl15@yandex.ru*

1. Problem actuality. The inverse problems of exploratory mathematical geophysics are well-spread in world. These problems are connected to necessity of search and workout of new geological fields, and also for estimation of remaining sources. Also these problems are interesting, being an incorrect problems of mathematical physics [1]. Such problems include the simplified case of the inverse gravimetric problem considered here. 2. Description of the mathematical model. 2.1. The straight problem includes: 1) Poisson's equation  $\Delta\eta = -4\pi G\psi$  with respect to the difference functions of gravitational potential  $\eta$  and solid density  $\psi$ , where  $\Delta\eta = \Delta(\varphi - \varphi_0)$ ,  $\psi = \rho - \rho_0$ , and values  $\varphi_0, \rho_0$  are considered as "normal values"; 2) boundary conditions on the potential difference function; 3) size of the target area  $\Omega$ . It is required to find the state function  $\eta = \eta(x, y)$ . 2.2. The inverse problem to this straight problem is as follows: Find the pair of numbers  $(a; b)$  so that the solution of problem (2.1) satisfies the boundary condition on the gradient of the gravitational potential along the upper boundary of the region. This follows to that gravimetric measurements are fulfilled along the Earth's surface. 3. Simplifications and assumptions of the problem: 1) the two-dimensional case (as a section of a certain spatial domain) is considered; 2) the normal values of the rock solid density and the gravitational potential in the search area  $W$  with an expandable boundary are known; 3) all areas are considered rectangular; 4) the dimensions of the target rectangle  $\Omega$  are known; 5) the coordinates of any point of the target rectangle  $\Omega$  are unknown. Thus, the problem is to restore its coordinates (one point is enough). 4. Functional derivative properties. The problem (2.2) reduces to the problem of minimizing the integral functional of the coordinates  $\mathcal{J}(a; b)$ , in which the function  $\eta$  is the solution of the direct problem (2.1). Construction of the Gateaux derivative by varying the functional  $\mathcal{J}$  with respect to the two coordinates  $a$  and  $b$  made clear the non-linearity of absolute value type by the direction  $h$  in obtained expression, and thus, there is non-differentiability by Gateaux of initial functional. This means that the standard approach which includes finding Gateaux derivative of functional, and then starting gradient method, wouldn't be applicable here. 5. It is planned in further to try here the subgradient method and genetic algorithms, to analyze the convergence of algorithms and compare them. Numerically, the problem is planned to solve in Python using cluster calculations.

### REFERENCES

1. *V.K. Khmelevskoy, V.I. Kostitsyn. The Basics of Geophysical methods // U-ty of Perm. - 2010, P. 47.*

**ON THE CONVERGENCE OF SOLUTION OF THE THREE-LAYER  
PERTURBED DIFFERENCE SCHEME TO SOLUTION OF ILL-POSED  
CAUCHY PROBLEM**

Sultanov M.A., Bakanov G.B.

*Akhmet Yassawi International Kazakh-Turkish University, Turkestan*  
*smurat-59@mail.ru, gbakan59@mail.ru*

The problem of convergence of the solution of a two-parameter family of a three-layer difference scheme to an exact solution of an abstract ill-posed Cauchy problem is considered. The proof of convergence is based on the concepts of stability of a difference scheme on functions with a finite support [1] and is based on obtaining a priori weight estimates of the Carleman type in the difference variant.

Let  $v$  be the solution of the following abstract ill-posed Cauchy problem

$$\frac{d^2 v}{dt^2} = Av + Kv, v(0) = v_0, v_t(0) = 0. \quad (1)$$

Here  $A = A_1 + A_2$ ,  $A_1 \geq 0$ ,  $A_2 \leq 0$  – linear, unbounded, independent of  $t$  operators with domain of definition  $D(A) \subseteq H$ ,  $H$  – Hilbert space; the operator  $K$  (perturbation operator) acts from the space  $L_2([0, T]; H)$  to  $L_2([0, T]; H)$ ; function  $v : [0, T] \rightarrow H$  is four times continuously differentiable in the strong sense, i.e.  $v \in C^4([0, T]; H)$ , and besides  $v(t) \in D(A)$  for all  $t \in [0, T]$ .

We assign to problem (1) a two-parameter family of difference schemes with two weights  $\sigma, q$ :

$$u_{t\bar{t}} - A_{1h} \left( \sigma \hat{u} + (1 - 2\sigma)u + \sigma \check{u} \right) - A_{2h} \left( q \hat{u} + (1 - 2q)u + q \check{u} \right) - \tilde{K} \check{u} = 0, \quad (2)$$

$$u(0) = g, \quad u_1 = u_0. \quad (3)$$

It is assumed that  $A_{1h}, A_{2h} \in \mathcal{L}(H_h)$ ,  $\tilde{K} \in \mathcal{L}(l_2(0, N; H_h))$ ,  $A_{1h} = A_{1h}^* \geq 0$ ,  $A_{2h} = A_{2h}^* \leq 0$ , the operators  $A_{1h}, A_{2h}$  commute and  $A_{1h}, A_{2h}, \tilde{K}$  approximate the operators  $A_1, A_2, K$ . In turn, the Hilbert space  $H_h$ , depending on the parameter  $h > 0$ , approximates the space  $H$ .

In this paper, we prove a theorem on the convergence of the solution of the difference scheme (2)-(3) to the solution of the ill-posed Cauchy problem (1), which is based on the stability theorem of the difference scheme [2].

This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (project N 3630 / GF4).

REFERENCES

1. *Bukhgeim A.L.* On the stability of difference schemes for Ill-posed problems // Reports of the Academy of Sciences of the USSR, Vol.270, N 1, P. 26-28.
2. *Sultanov M.A.* Estimates of the stability of solutions of a three-layer difference scheme with two weights for ill-posed Cauchy problems // Siberian Electronic Mathematical Reports. 2015, Vol.12, P. 28-44.

## NUMERICAL METHOD FOR SOLVING TWO DIMENSIONAL FREDHOLM INTEGRAL EQUATIONS

Temirbekova L.N.

*KazNPU named after Abai, Almaty, Kazakhstan*

*laura-nurlan@mail.ru*

The work is considered numerical method of solving inverse problems for two dimensional coefficient hyperbolic equations. The coefficient inverse problem solved using the method of the two dimensional Gelfand-Levitan integral equation [1]. To numerically solve the two dimensional Gelfand-Levitan integral equation, which is the Fredholm integral equation of first kind, we use effective parallel algorithms [2-3].

### REFERENCES

1. *Gelfand I.M., Levitan B.M.* On the determination of a differential equation from its spectral function, Journal, *Izv. Akad. Nauk USSR. Ser. Mat.*, V.15, 309-360 (1951).
2. *Kabanikhin S.I.* Inverse and Ill-Posed Problems, Siberian Scientific publishers, Novosibirsk, (2009).
3. *Romanov V.G.* Inverse Problems of Mathematical Physics, Nauka, Moscow, (1984).

## SEISMIC RISK ESTIMATION OF ALMATY CITY USING GIS ITRIS

Turarbek A.T., Begadil Z., Kozhakenov G.  
*Al-Farabi Kazakh National University, Almaty*  
*turarbekasem1@gmail.com*

The most seismically dangerous territory in Kazakhstan is the Northern Tien Shan, which is located near to Almaty and cover it. The probability of appearing powerful earthquakes in Almaty region is high, because in the course of the seismic process there is a periodicity of alternating stages of activation and calm [1].

Catastrophic earthquakes cause irreversible changes in the geological environment and have pernicious consequences for industrial and civil objects. In this regard, seismic risk estimation is necessary, i.e. to develop a problem related to the seismic estimation of impacts on industrial and civil structures, economic and social damage due to earthquakes, as well as ensuring the city life. Seismic risk is probability of social and economic damage associated with earthquakes in the considerable territory for the certain period of time. With regard to the Republic of Kazakhstan, the question for any of the settlements, including Almaty, has not yet been fully considered.

At the moment, with full confidence we can say that in the case of a strong earthquake in Almaty may experience some uncontrollable emergency situation, as the administrative authorities and the control bodies cannot fully predict the course of events. This is due to the fact that the city authorities do not have objective data on the seismic risk of such a large metropolis as Almaty.

Geo-information system named ITRIS allows to perform predictions of earthquake consequences within the territory exposed to seismic action, and with its help it is possible to carry out simulation and calculate the damage [2]. To determine the accuracy of possible consequences calculations of earthquakes using ITRIS GIS, the first stage catastrophic earthquakes of seismic activity simulations was carried out, as a result almost matched the actual data with outcomes [3].

### REFERENCES

1. Sadykova A.B. Seismic danger of the territory of Kazakhstan, 2012, Almaty, P. 344.
2. Kabanikhin S.I., Krivorotko O.I., Marinin I.V.. Three-dimensional GIS analysis and assessment of natural and man-made disasters. Preliminary operational analysis and assessment of the consequences of natural and man-made emergencies, 2013, Novosibirsk, .
3. Krivorotko O.I., Kabanikhin S.I., Turarbek A.T., Betmesov M.A., Marinin I.V., Sadykova A.B.. Geoinformation system of Kazakhstan. Mathematical models of geoinformation system of Kazakhstan // Proceedings of the International Conference on Computational and Applied Mathematics "VPM'17" in the framework of "Marchuk Scientific Readings Novosibirsk, June 25 - July 14 [Electron. resource]. <http://conf.nsc.ru/cam17/en/proceedings>. P. N455-N462.

**THE SOLUTION OF THE INVERSE PROBLEM OF DETERMINING  
PARAMETERS FOR THE MATHEMATICAL MODEL OF HIV DYNAMICS**

Yermolenko D.V., Krivorotko O.I., Kabanikhin S.I.

*Novosibirsk State University, Novosibirsk,*

*Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk  
ermolenko.dasha@mail.ru, olga.krivorotko@sccc.ru, kabanikhin@sccc.ru*

Mathematical models of immunological diseases are described by systems of nonlinear ordinary differential equations and characterized by a set of parameters. These parameters describe individual features of immunity and diseases of the patient. It is necessary to find the set of parameters for constructing an individual treatment plan.

In this paper, the inverse problem [1] of determining the HIV-infection parameters and the immune response using additional measurements at fixed times for the mathematical model of HIV dynamics [2] has been investigated numerically. The stability of the inverse problem solution is analyzed using the singular value decomposition for linearized matrix of the inverse problem. The state variable observations are different from each other by orders of magnitude, intuitively, it is critical that the estimation scheme take this into account. One way to do this is by appropriately weighting the states in a least squares cost criterion. A genetic algorithm [3] for solving a least squares minimization problem on iteration of least squares method is investigated. To determine the level of error in the solution of the inverse problem, the confidence intervals of all parameters are obtained and analyzed.

After determining the model parameters of the, the problem of choosing the optimal treatment for a particular patient is investigated. The problem of optimal treatment control is reduced to the problem of minimizing misfit function that characterizes combination of viral load and treatment costs. To find the continuous optimal treatment control, the Pontryagin's Maximum Principle is used.

*The work has been supported by the grant no. 1746/GF4 of the Ministry of Education and Science of Republic of Kazakhstan, Ministry of Education and Science of Russian Federation and by the Scholarship of the President of RF no. MK-1214.2017.1.*

REFERENCES

1. S.I. Kabanikhin, Inverse and Ill-Posed Problems: Theory and Applications, Berlin/Boston: de Gruyter, 2012.
2. B.M. Adams, H. T. Banks, et al., HIV dynamics: Modeling, data analysis, and optimal treatment protocols, *Journal of Computational and Applied Mathematics*, 184, 10:49 (2005).
3. Kabanikhin S.I., Krivorotko O.I., Yermolenko D.V., Latyshenko V.A., Kashtanova V.N. Inverse problems of immunology and epidemiology // *Eurasian Journal of Mathematical and Computer Applications*. 2017. V.5, Issue 2. P.14-35. ISSN 2306-6172.



## ABOUT ONE INVERSE PROBLEM FOR OCEANOLOGY

Zakirova G.A., Kadchenko S.I.  
*South-Ural state university, Chelyabinsk*  
*zakirovaga@susu.ru, kadchenkosi@susu.ru*

The results obtained in [1] allowed to construct an efficient numerical method to investigate mathematical models with perturbed discrete lower semibounded operators.

Consider the problem of reconstructing the distribution plots of water in the deep ocean by the phase characteristic of the internal waves at the free surface. The boundary value problem

$$\begin{aligned} W''(z) - \frac{\mu(z)}{g}W'(z) + \frac{\mu(z) - \omega^2}{\omega^2 - f^2}k^2W(z) &= 0, 1) \quad -H < z < 0, \\ W(-H) = 0, \quad W'(0) - \frac{gk^2}{\omega^2 - f^2}W(0) &= 0, \\ \mu(z) &= -\frac{g}{\rho_0}\rho'_0(z) \end{aligned} \quad (1)$$

describes the free vibrations of a stratified ocean in the Boussinesq approximation and "solid" cap for the amplitude of the vertical oscillations of liquid particles. Reduce (1) to inverse spectral problem (2)

$$\begin{aligned} \frac{\omega^2 - f^2}{\mu(z) - \omega^2}W''(\xi) &= \beta W(\xi), \quad 0 < \xi < 1, \\ W(0) &= W(1) = 0, \end{aligned} \quad (2)$$

In (1) and (2)  $W(z)$  is the function of the amplitude of oscillations of fluid particles in the direction of the axis  $Oz$ ;  $\mu(z)$  – the square of the buoyancy frequency;  $\rho_0(z)$  – the density of the fluid in the equilibrium state;  $g$  – gravitational acceleration;  $\omega$  – circular frequency of free oscillations of a heterogeneous liquid;  $k$  – the wave number of vertical particle oscillations of inhomogeneous liquid, corresponding to given frequency;  $f = 2\Omega \sin \varphi$  – Coriolis parameter;  $\varphi$  – latitude of location;  $H = const$  – water depth,  $\beta = -H^2k^2$ .

To solve the problem (2) the algorithm for numerical solution of inverse spectral problems for perturbed semibounded discrete operator is proposed. This algorithm allows to restore the function  $\mu(z)$  in the nodes of the discretization and to determine the required density of water  $\rho_0$ .

The work has been supported by Act 211 Government of the Russian Federation, contract No 02.A03.21.0011

### REFERENCES

1. *Kadchenko S.I., Zakirova G.A.* Computation of eigenvalues of discrete lower semibounded operators // «Applied Mathematical Sciences», 2016, vol.10, № 7, pp. 323–329.

## THE BLOCKCHAIN HYPE: CURRENT RESEARCH STATE OF TECHNOLOGY

Zhumat F.S., Duisebekova K.S.

*International Information Technology University, Almaty*

*zhumat.fariza@gmail.com*

### Abstract

Cryptocurrencies such as bitcoin may have captured the public's fancy and also a healthy dose of skepticism, but their underlying blockchain technology is proving to be of practical benefit to organizations. The reason for the interest in Blockchain is its central attributes that provide security, anonymity and data integrity without any third party organization in control of the transactions, and therefore it creates interesting research areas, especially from the perspective of technical challenges and limitations. Many industries are exploring its benefits and testing its limitations, they believe in the ability of blockchain technology to improve efficiency in such things as the trading and settlement of securities. This article tackles a comprehensive overview of the last update in this field and briefly investigates recently proposed decentralized systems with their solutions based on blockchain technology. The methodology is a mapping study that provides an overview of blockchain based technology research area. The objective is to understand the current research topics, challenges and future directions regarding Blockchain technology from the technical perspective. The conclusion that has been drawn represents a statistical data on the expansion and application trends in the blockchain technology. The main target of this article is to give nearly full image of today's blockchain technology and the related fields with brief details.

*Keywords:* blockchain, decentralized system, current state, applications

**The use of digital educational resources in teaching ill-posed system of linear algebraic equations**

Bidaibekov Y.I., Sholpanbaev B.B., Akimzhan N.Sh.  
*Abai Kazakh national pedagogical university, Almaty*  
*nagima\_akim@mail.ru*

One of the requirements for higher education is the fundamental training of students. The realization of this requirement can be the design of such content of training that corresponds to the modern achievements of world science, has didactic and methodological significance, the logic of the presentation of the educational material, the unity of the teaching material, generality, completeness, optimality and other criteria. Obviously, the content of teaching students inverse and ill-posed problems (IIP) should meet the criteria listed above, contain not only theoretical material, but also a system of teaching tasks and examples, and also the principles of teaching IIP. When developing appropriate criteria for the selecting content of teaching IIP the research of A.E. Abylkasymova, S.I. Arkhangelsky, R.M. Aslanov, M.A. Bektemesov, O.A. Belyak, E.Y. Bidaybekov, G.D. Bukharov, A.O. Vatulian, A.M. Denisov, V.I. Zagvyazinsky, S.I. Kabanykhin, V.S. Kornilov, V.S. Lednev, E.U. Medeuov, A.G. Mordkovich, E.I. Smirnov, Zh. Suleimenov, S.E. Temirbolat, G.G. Khamov, and others were used. The following criteria are singled out in the mentioned studies: the unity of the educational material and the content lines; basic knowledge, skills, skills and methods; logical spiral; generality; completeness; optimality and a number of others[1].

One of the sections of the content of teaching IIP is the section of ill-posed problems of linear algebra. In this section, much attention is paid to solving systems of linear algebraic equations (SLAE). This is dictated by the fact that in the process of constructing computational algorithms for solving a wide class of applied problems one has to deal with the search for solutions of SLAE. Finding solutions to SLAE is often an ill-posed problem, for example, when its matrix is ill-conditioned. This requires the choice of an effective method for solving SLAE. The presence of such problem situations requires the choice of an effective method for solving SLAE [2].

When teaching incorrect SLAE, it is advisable to use the digital educational resource (RED) "Systems of linear algebraic equations" , which encompasses a lecture, seminar lesson and independent work of students. And the existing glossary and additional material allows studying this material more deeply.

REFERENCES

1. *Bidaybekov E.I., Kornilov V.S., Kamalova G.B.* Inverse Problems for differential equations in education // Inverse Problems: Modeling and Simulation (IPMS-2014): Abstracts of the 7th International conference, 2014, Fethiye, Turkey, P. 69.
2. *Kabanikhin S.I.* Inverse and ill-posed problems: textbook for university students - Novosibirsk: Siberian scientific publishing trust, 2009.– 458 p.