Fracture closure pressure evolution prediction in water-injection wells using hydrogeomechanical modeling

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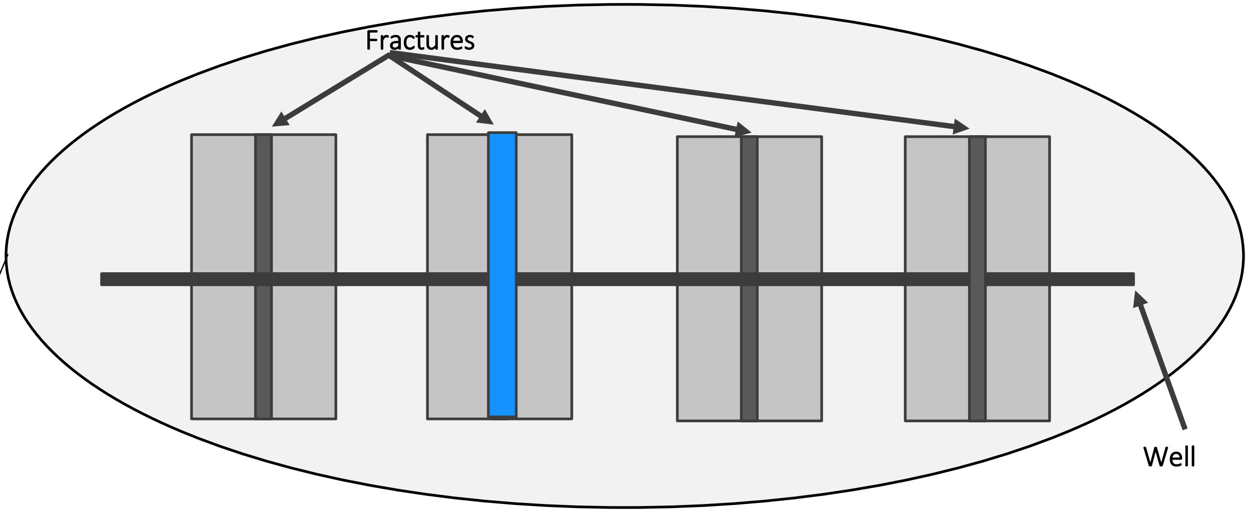
Content:

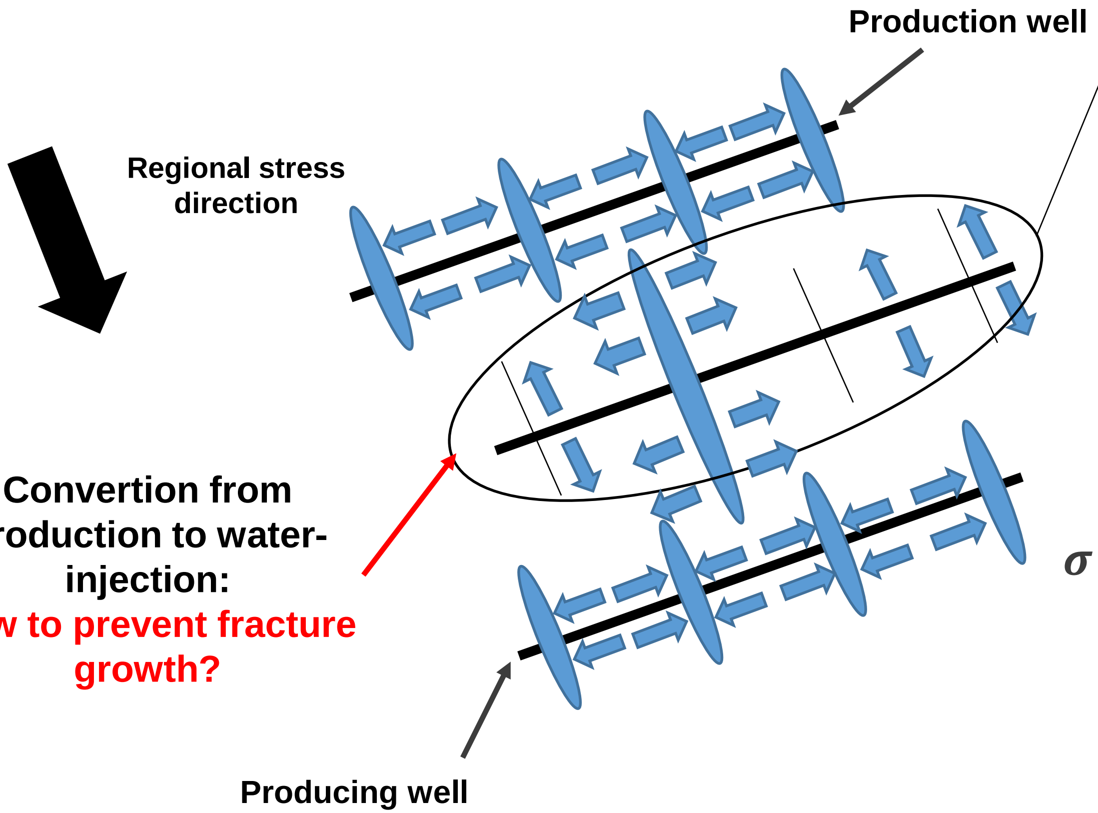
- **The influence of pore pressure on the effective stresses. Possible effects**
- **Coupled hydro-geomechnical model**
- **1D poroelastic horizontal strain model and its comparison with coupled hydro-geomechnical model**
- **Semi-analytical model and its comparison with coupled hydro-geomechnical model**
- **Coupled hydro-geomechnical model for real case**

Goals:

Goal: to prevent the growth of fractures in the injection well

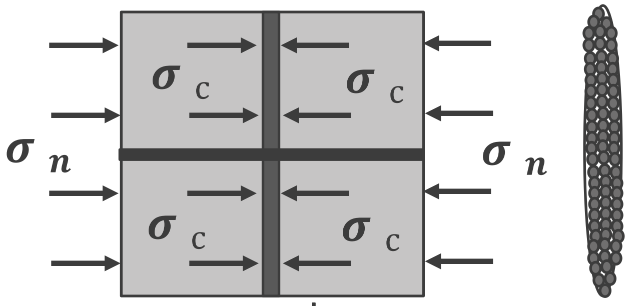
- **Solving the problem of pressure distribution**
- **Solving the problem of poroelasticity**
- **Searching for fracture propagation criteria**





Convertion from production to water-injection:
How to prevent fracture growth?

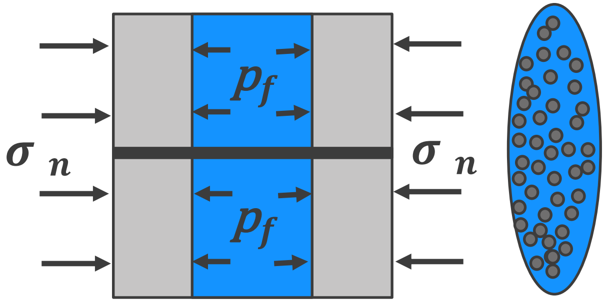
Schematic Illustration of closed fracture



$\sigma_n = \sigma_f + p_f$

$|\tau| \leq \eta \, \sigma_c + S_0$

Schematic Illustration of open fracture



$\sigma_n = p_f$

$\tau = 0$

The influence of pore pressure on the effective stresses: Stress path

- **Changes in effective vertical and horizontal stresses with a change in pore pressure are not proportional.**

«Stress path» for total stress:

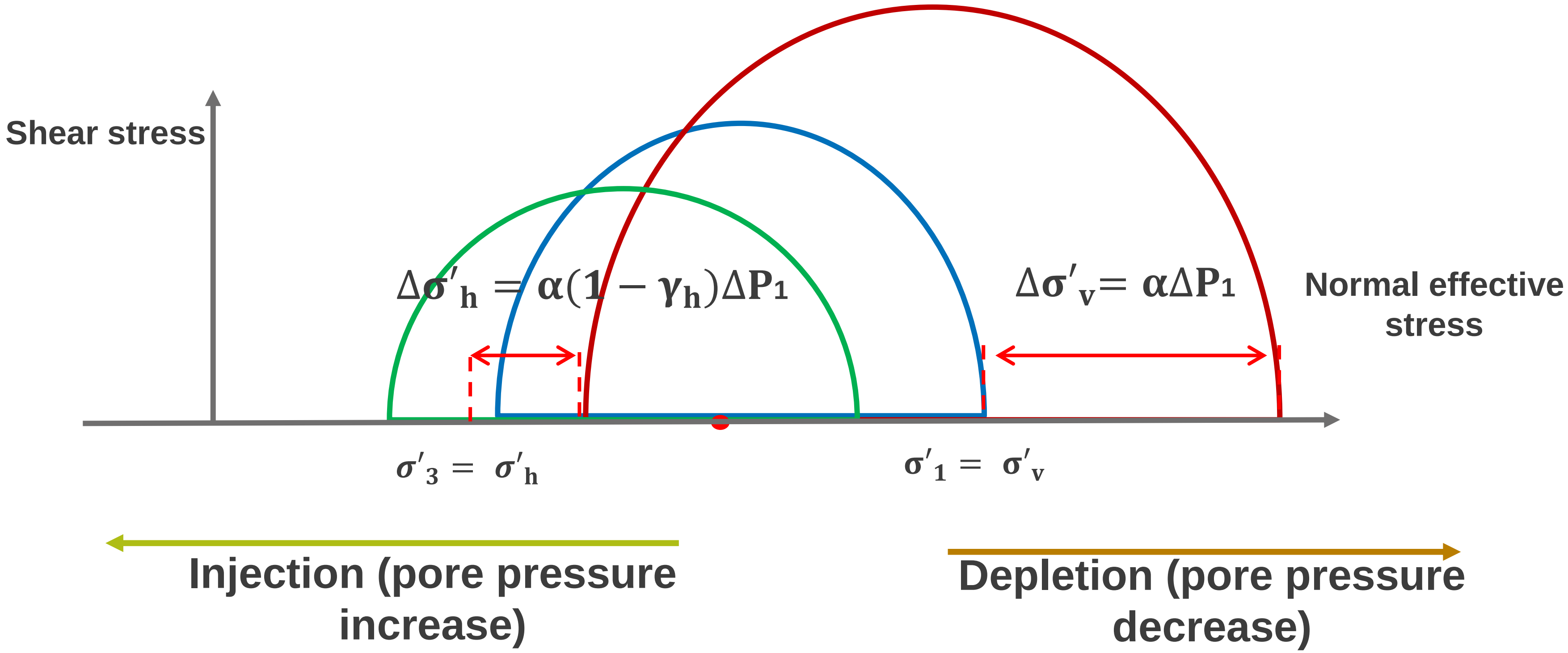
$\gamma v = \frac{\Delta\sigma_v}{\Delta P} = 0$ - vertical

$\gamma h = \frac{\Delta\sigma_v}{\Delta P} = \alpha\frac{1-2\vartheta}{1-\vartheta}$ - horizontal

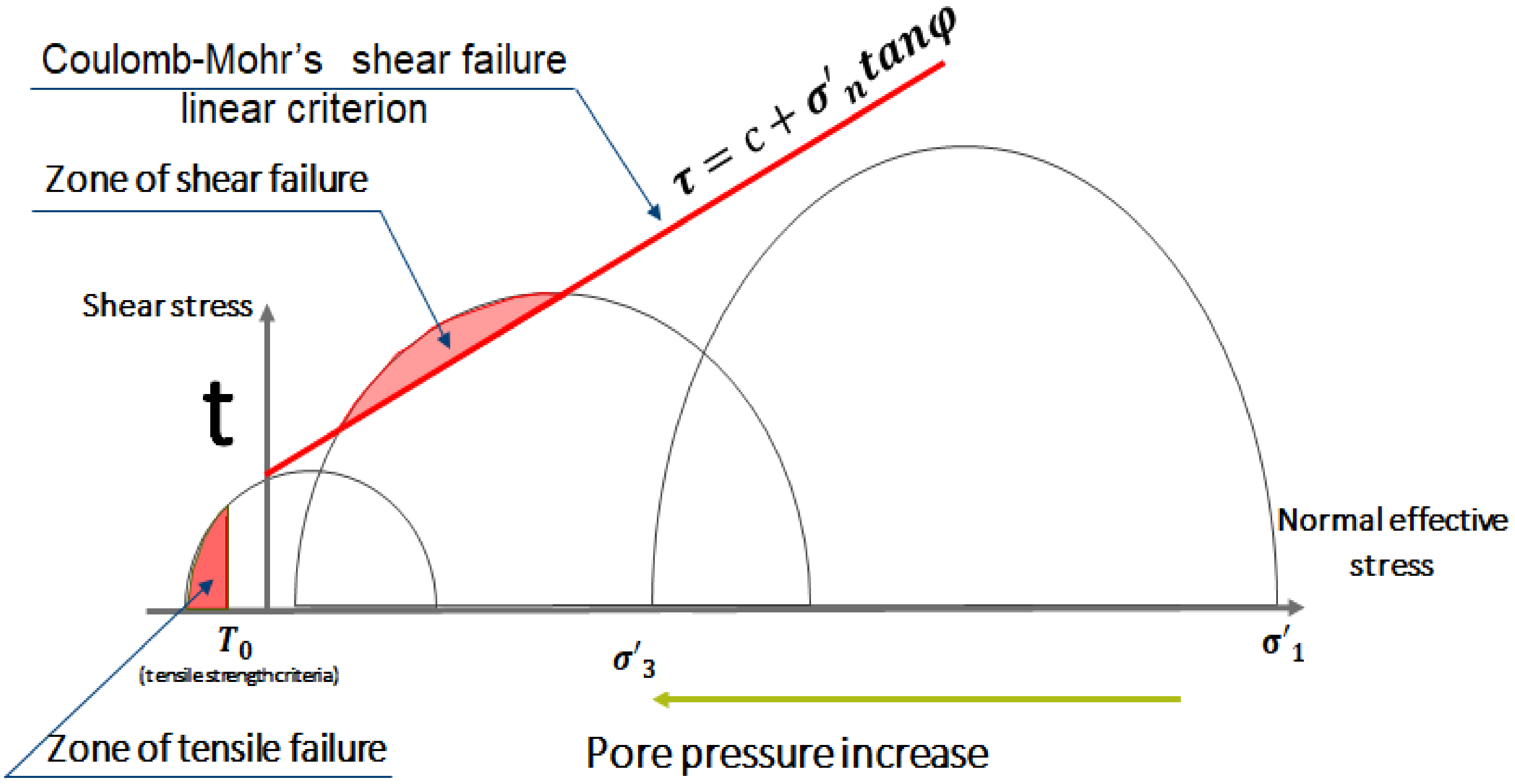
«Stress path» for effective stress:

$$\frac{\Delta\sigma'_v}{\Delta P} = \alpha$$

$$\frac{\Delta\sigma'_h}{\Delta P} = \alpha(1-\gamma_h)$$



The influence of pore pressure on the effective stresses : failure criteria



Mathematical model

General equations:

$$\sigma_{ij} = \lambda \varepsilon_{vol}\delta_{ij} + 2G\varepsilon_{ij} - C\zeta\delta_{ij}$$

$$\sigma'_{ij} = \sigma_{ij} - \delta_{ij}\alpha\sigma_f,$$

Hydrodynamical model:

Boundary conditions:

$$k\frac{k_{ri}}{\mu_i}(\nabla p_i - \rho_i g \nabla z)n|_{\Gamma} = 0$$

$$p_i|_{\Gamma} = p_i^0(\Gamma, t)$$

Initial conditions:

$$p(x,0) = p_0(x),\ x \in \Omega \backslash \{\cup_i \Gamma_{f_i}\}.$$

Geomechanical model:

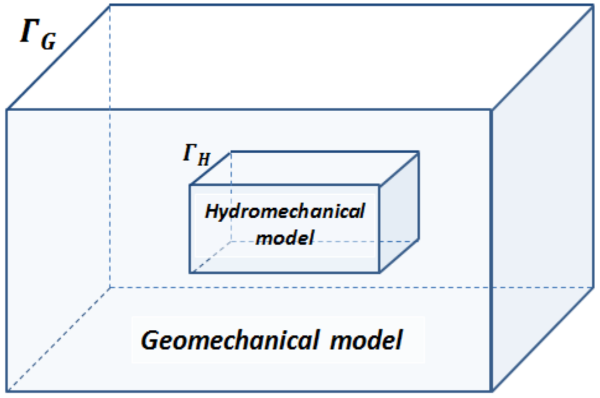
Boundary conditions:

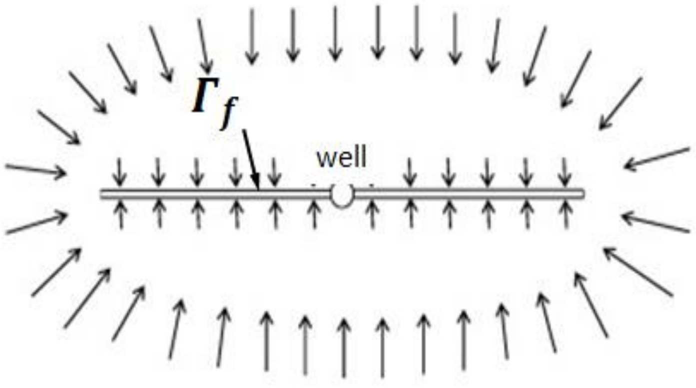
$$u = 0,\ x \in \Gamma_{bot}$$

Initial conditions:

$$\sigma_n = 0,\ x \in \Gamma_{top}$$

$\sigma = \sigma_0,\ x \in \Gamma_{side}$.ll:





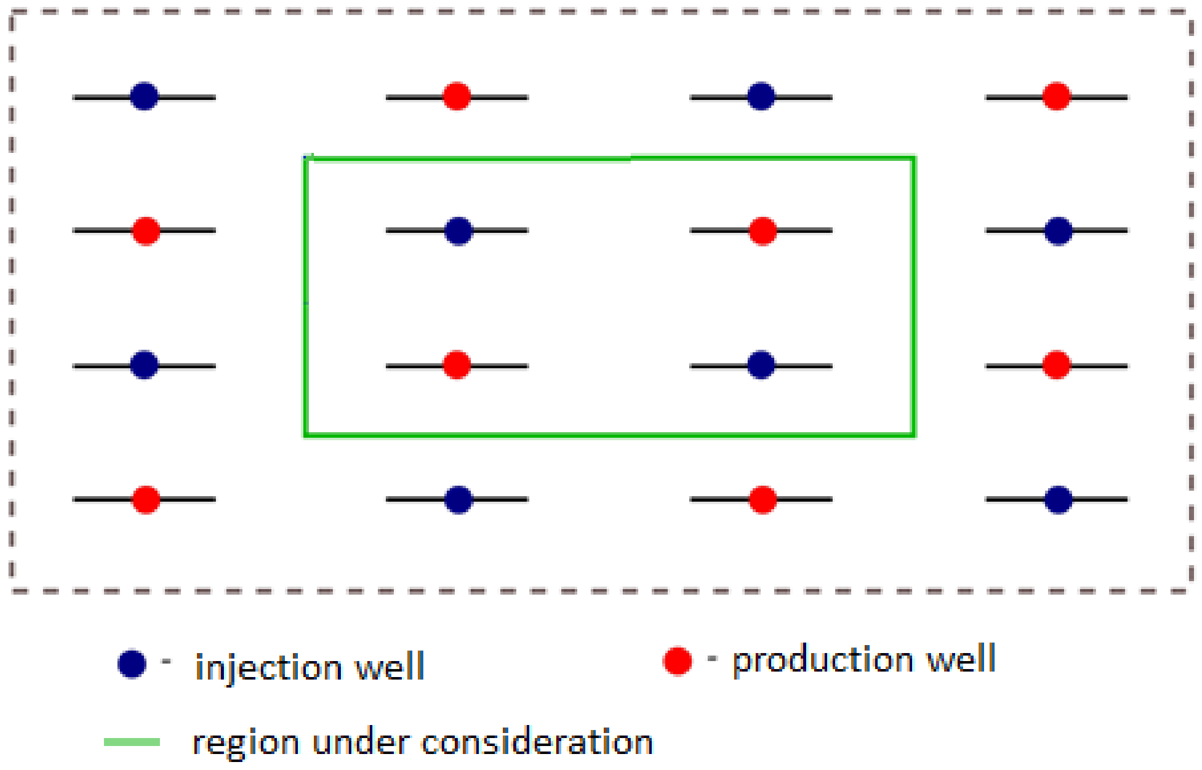
Assumptions in model:

- Reservoir is isothermal.
- The presence of proppant in the fracture is not taken into account.
- Interaction between neighboring fractures is not taken into account.

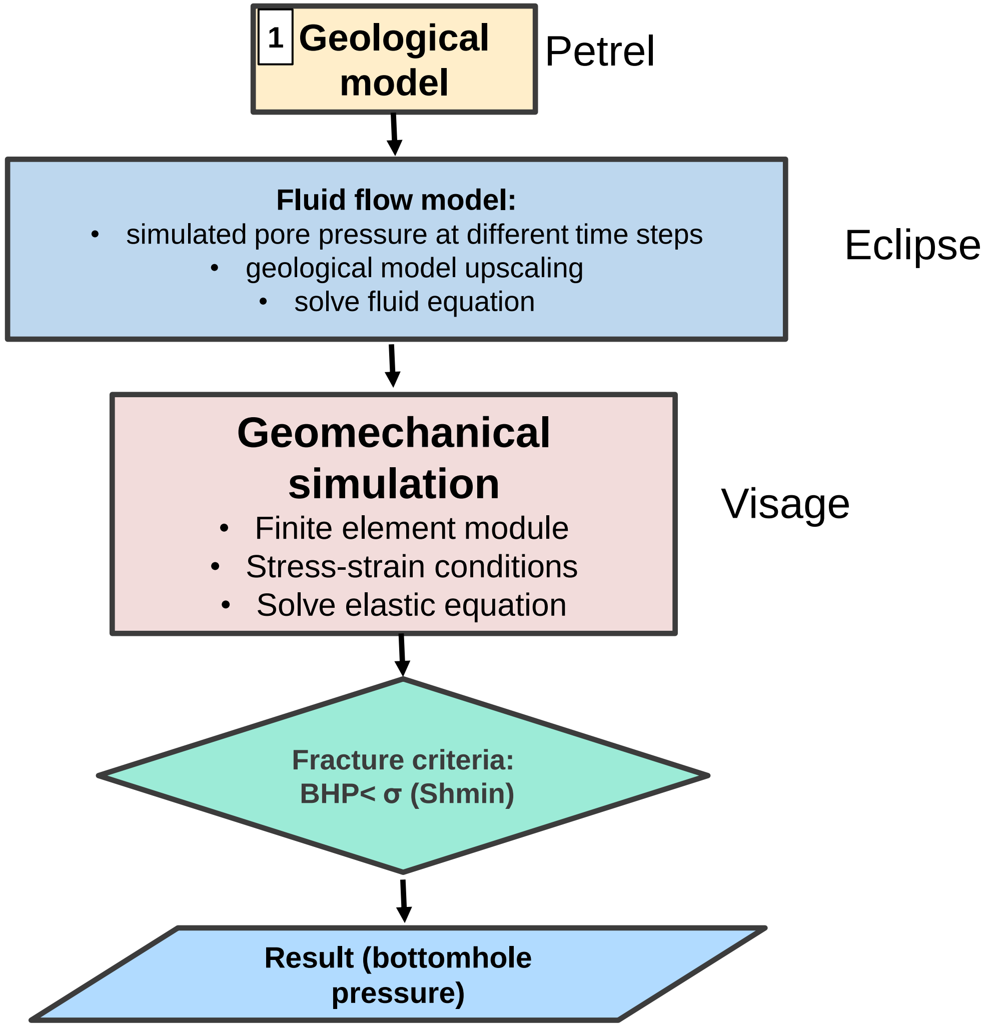
First case (ideal case) – symmetry and no anisotropy

CONDITIONS:

- **Linear periodic system.**
- **Region under consideration is a rectangular element with 2 injection and 2 production wells with hydraulic fracturing.**



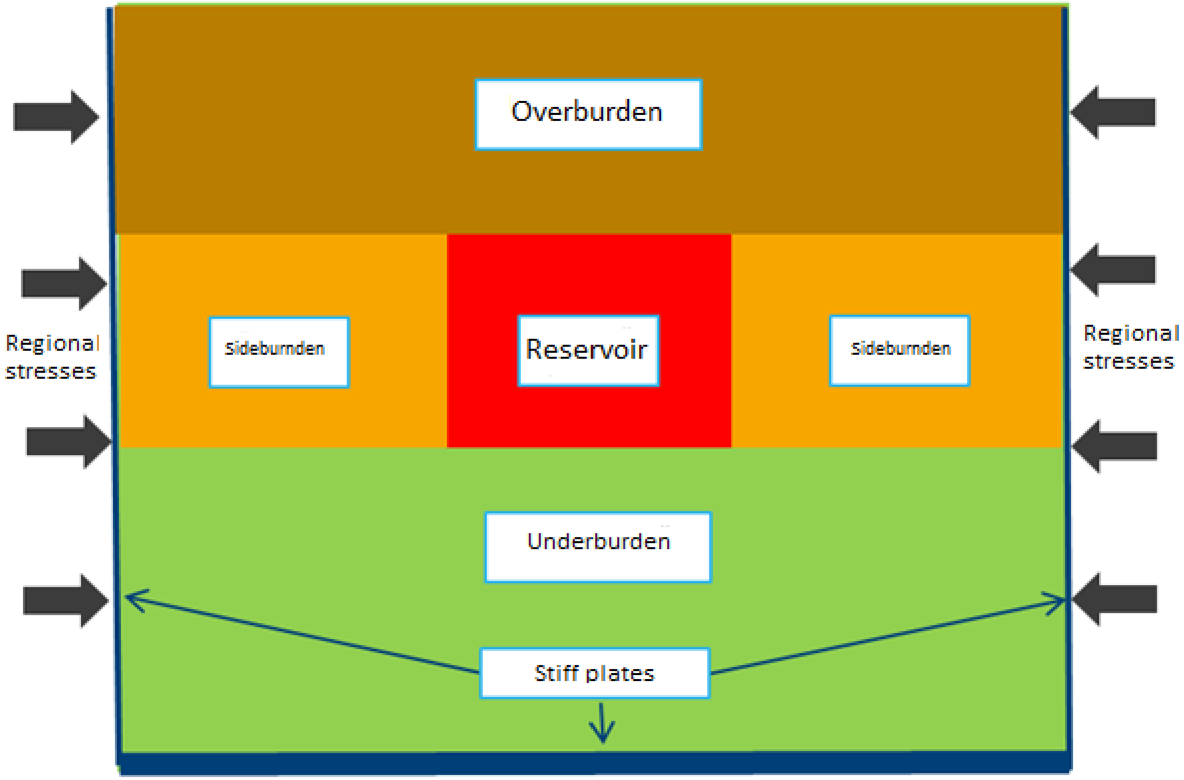
Algorithm of coupled hydro-geomechnical model construction



Geomechanical model

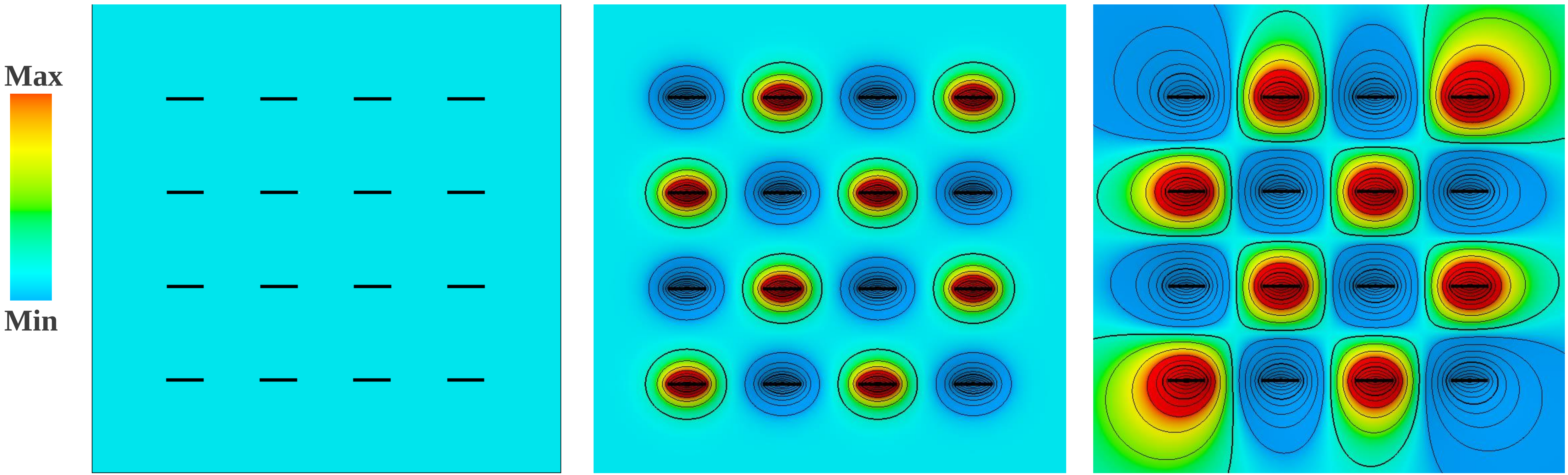
Calculation parameters:

Parameters/ intervals	Poisson's ratio	Young modulus, GPa	Bulk density, g/cm3	Biot coeff.
Overburden	0.3	15	2,1-2,4	0.8
Reservoir	0.27	19	2,4	0.8
Sideburden	0.3	15	2,5-5	0.8
Stiff plates	0.2	60	5	0.8



Schematic illustration of geomechanical model

Results of hydrogeomechanical modeling: pore pressure



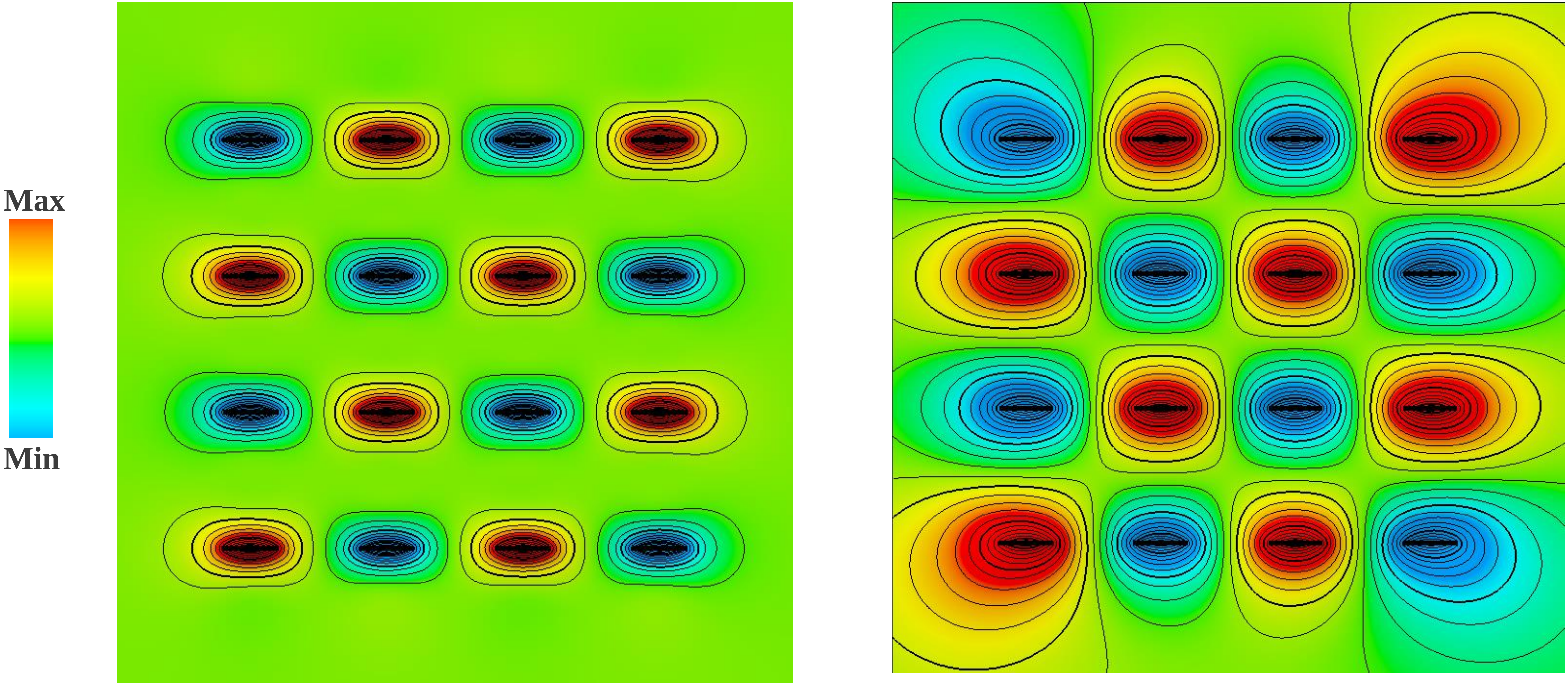
Initial time step

Middle time step

Last time step

fracture

Results of hydrogeomechanical modeling: σ_{xx}

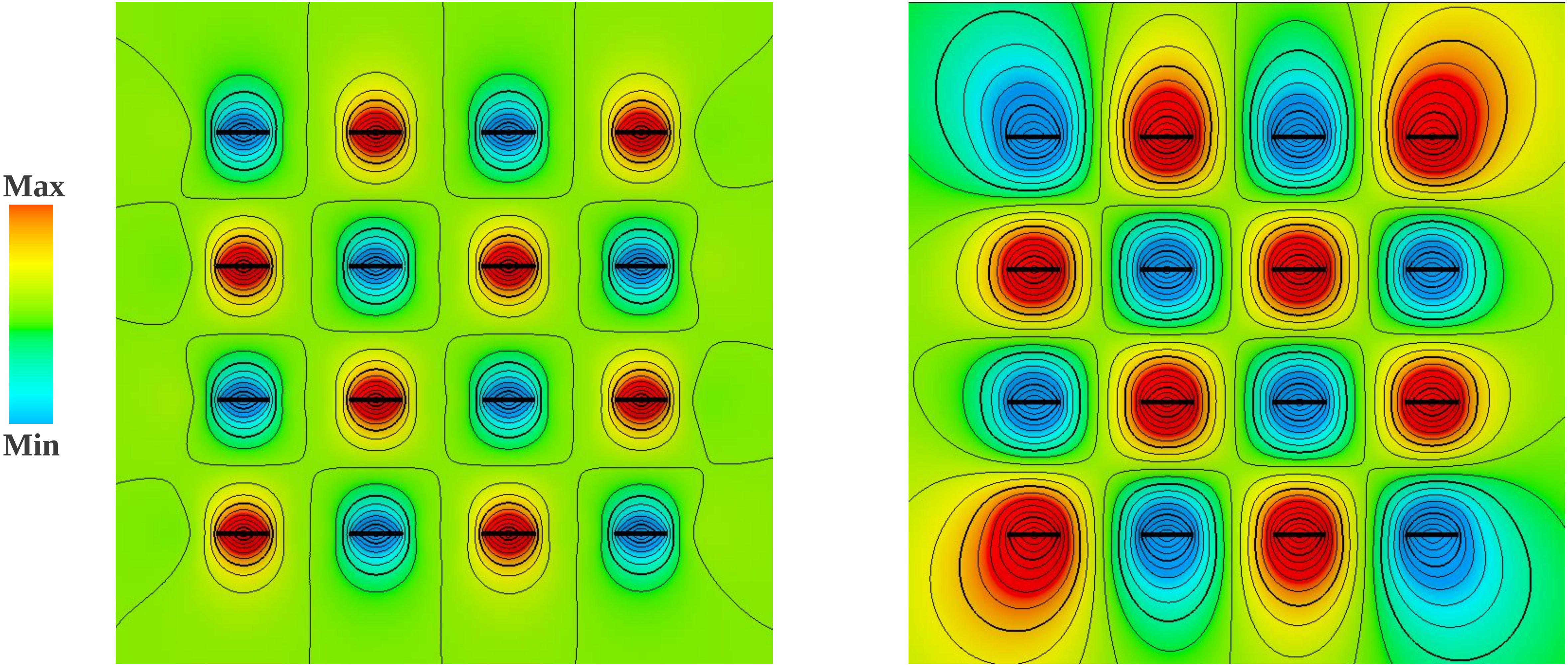


Middle time step

Last time step

fracture

Results of hydrogeomechanical modeling: σ_{yy}

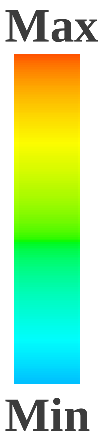


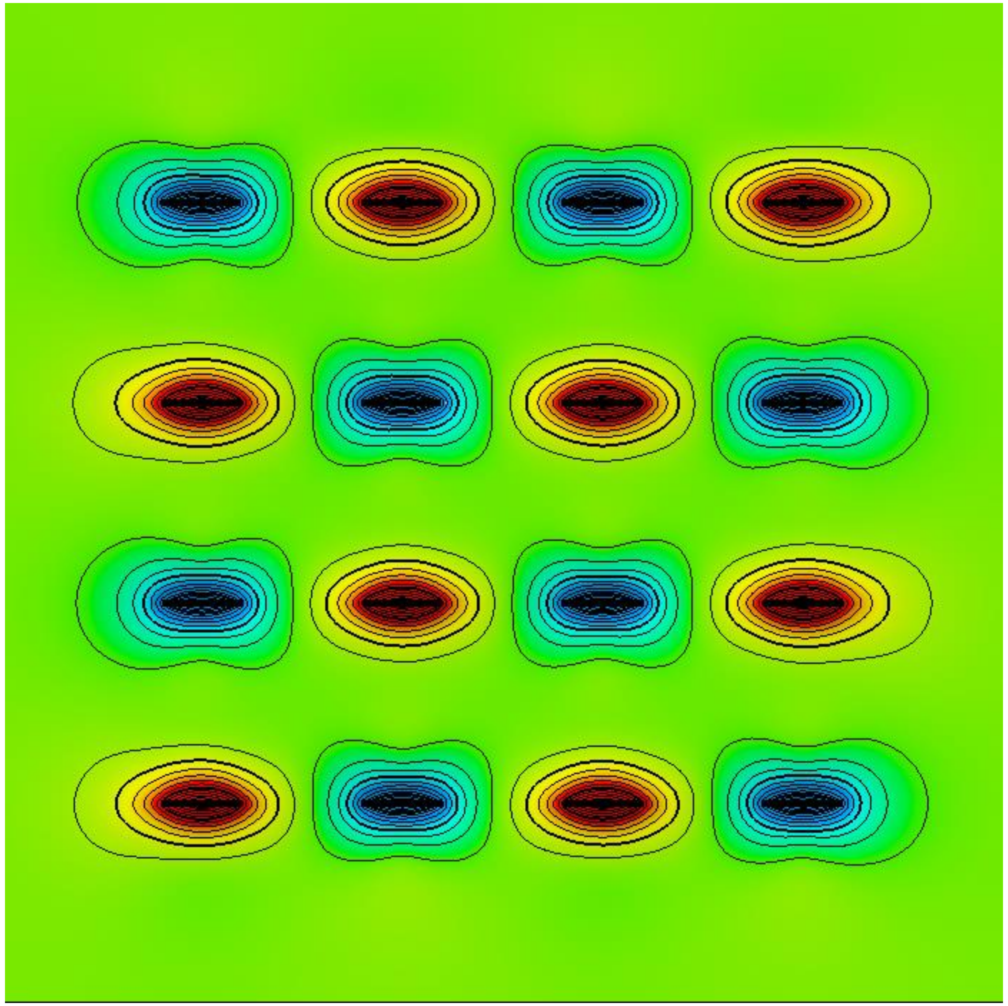
Middle time step

Last time step

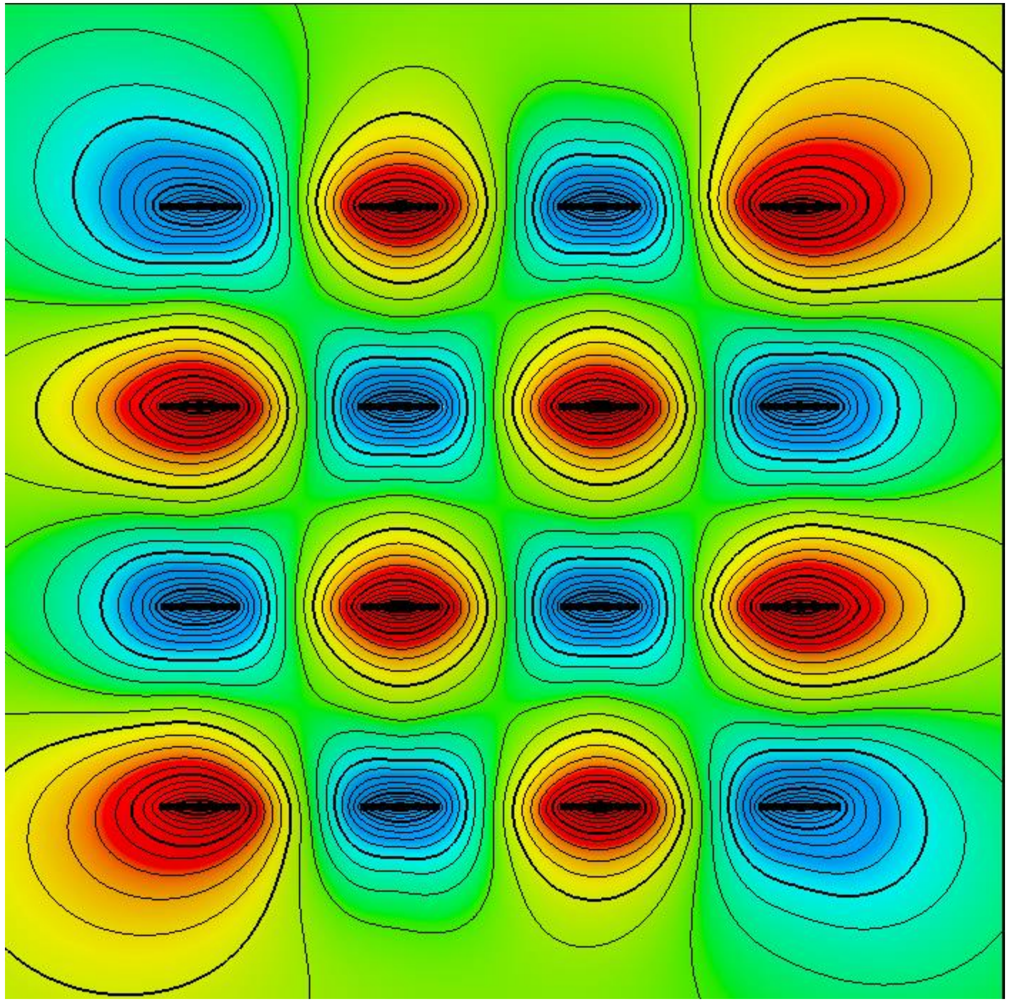
fracture

Results of hydrogeomechanical modeling: σ_3





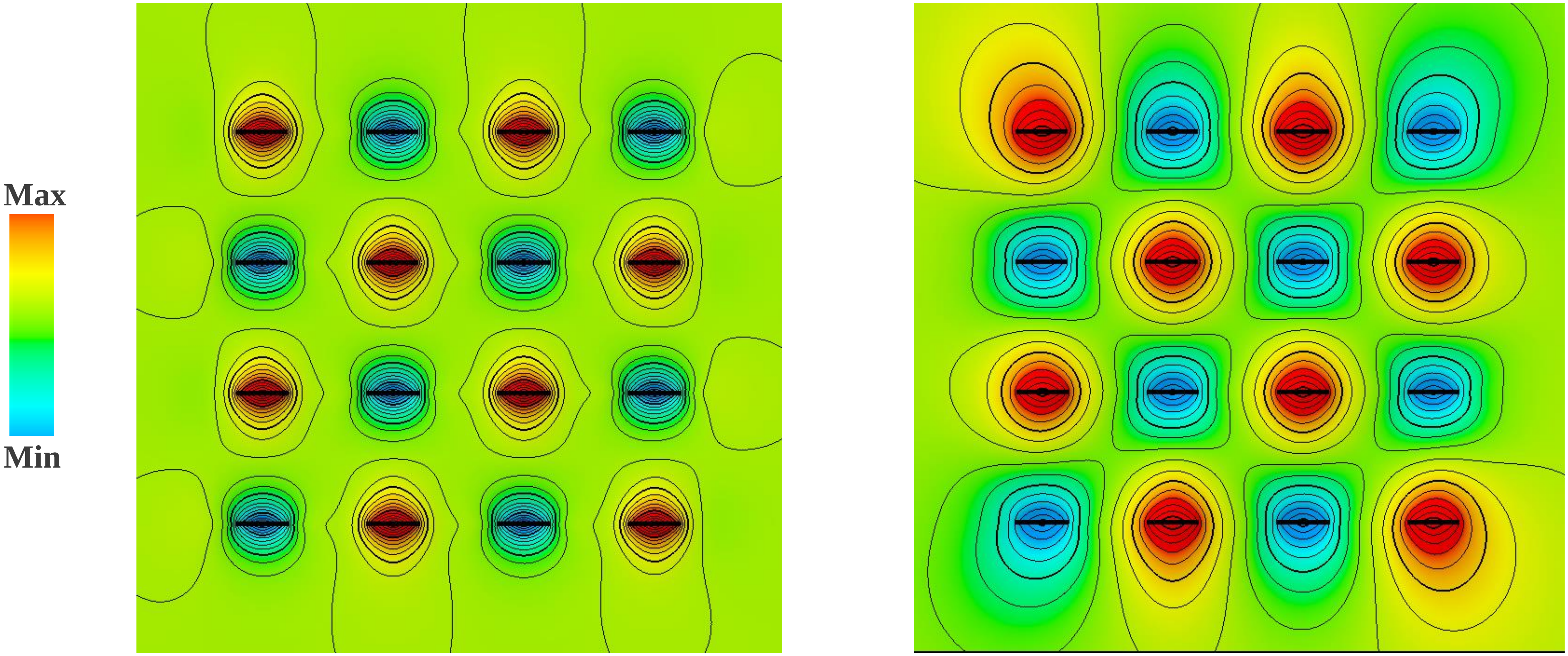
Middle time step



Last time step

— fracture

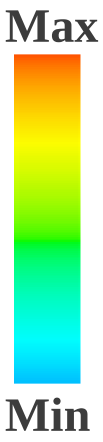
Results of hydrogeomechanical modeling: $\sigma_{3\mathrm{eff}}$

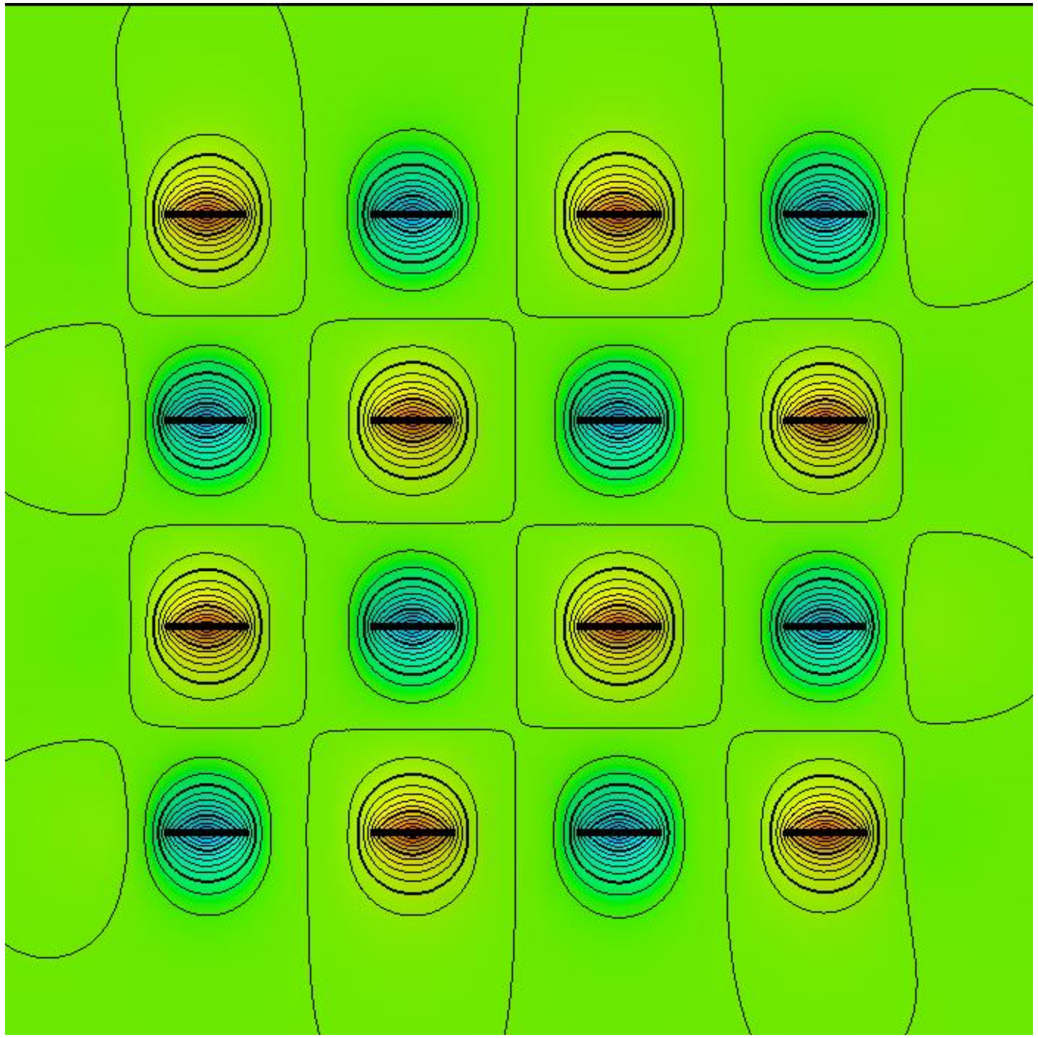


Middle time step **Last time step**

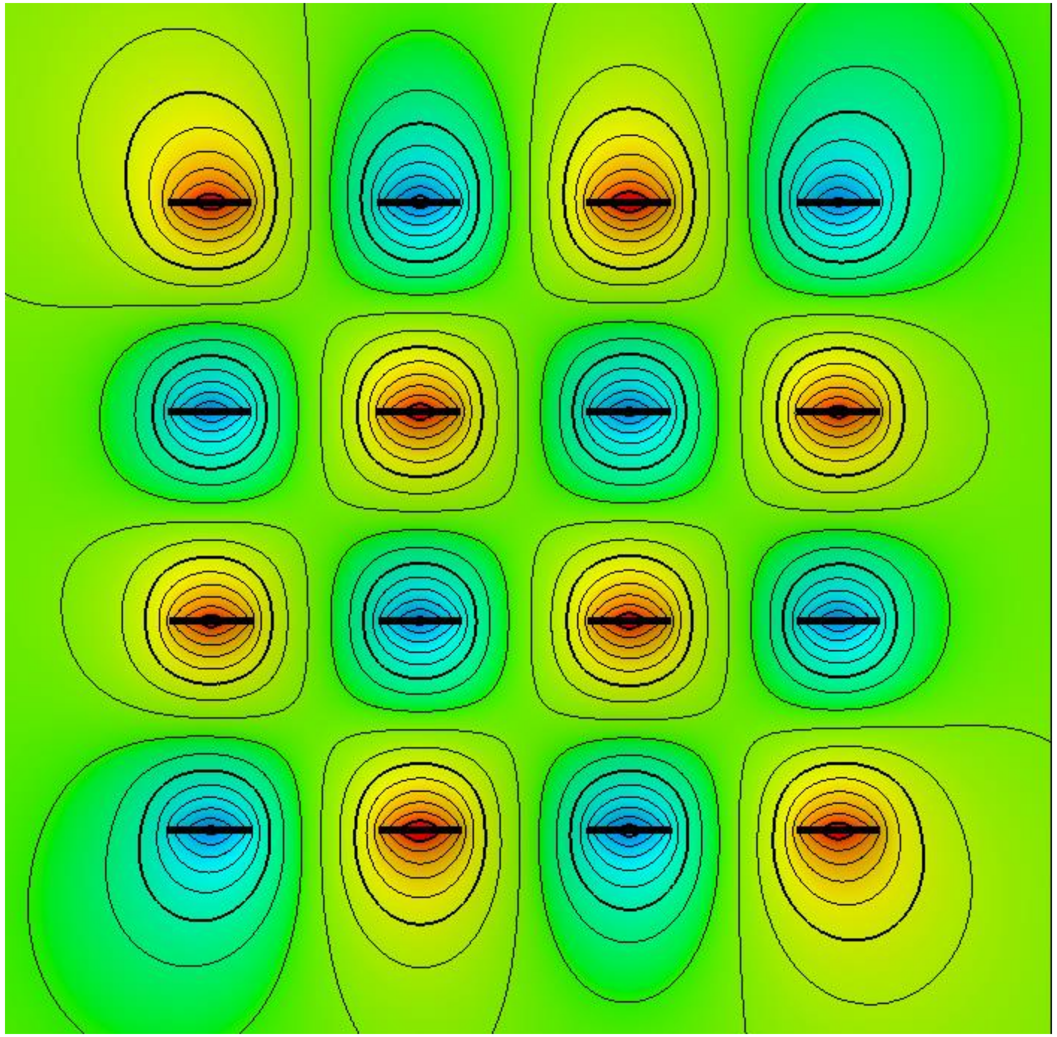
fracture

Results of hydrogeomechanical modeling: $\sigma_{xx\text{eff}}$





Middle time step

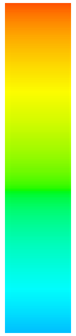


Last time step

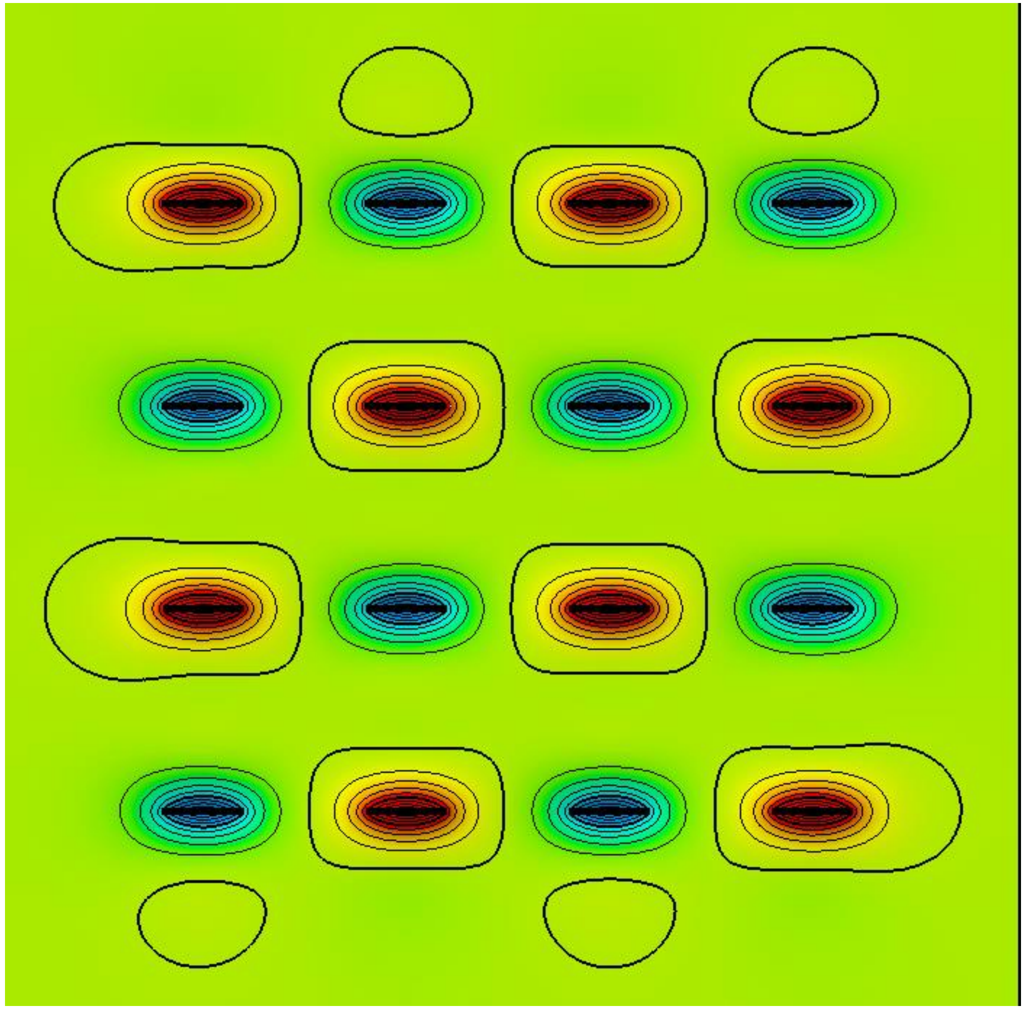
fracture

Results of hydrogeomechanical modeling: $\sigma_{yy\mathrm{eff}}$

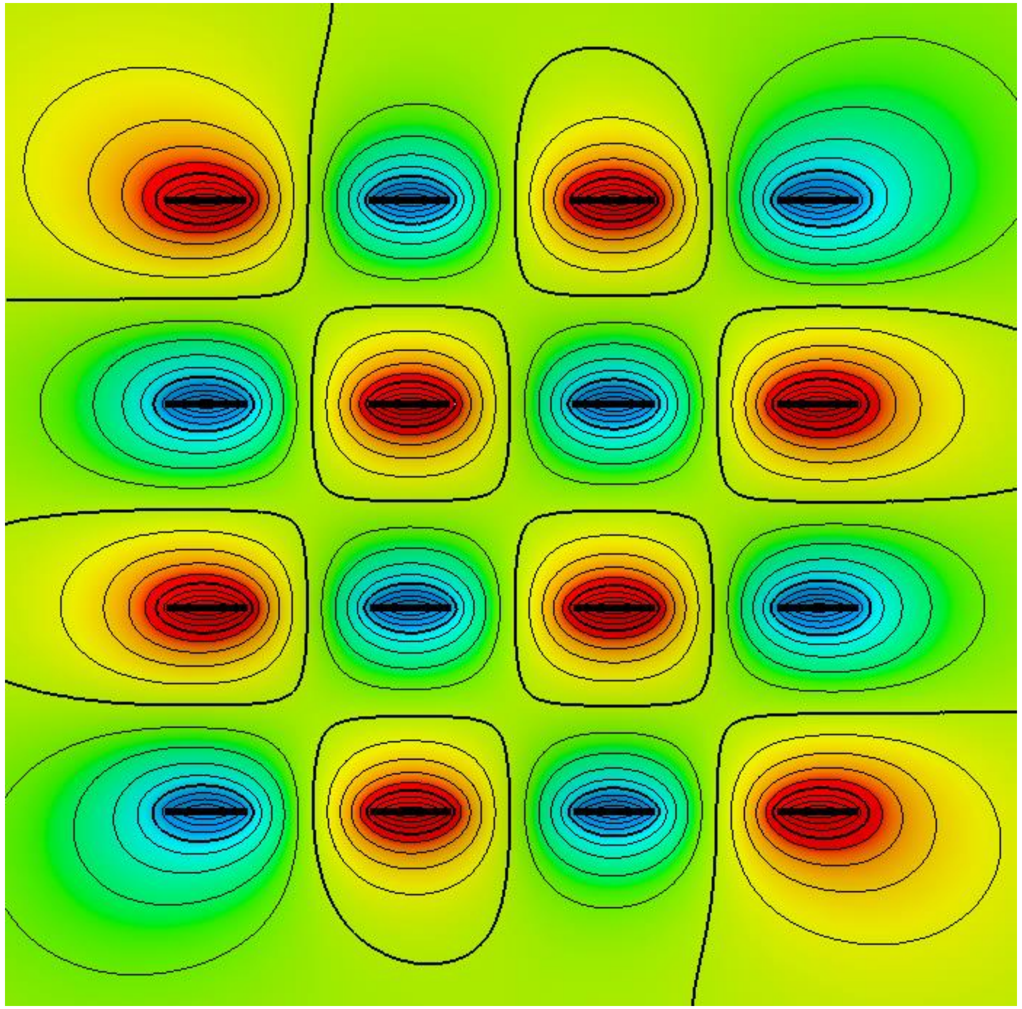
Max



Min



Middle time step

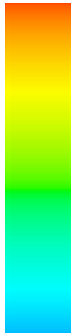


Last time step

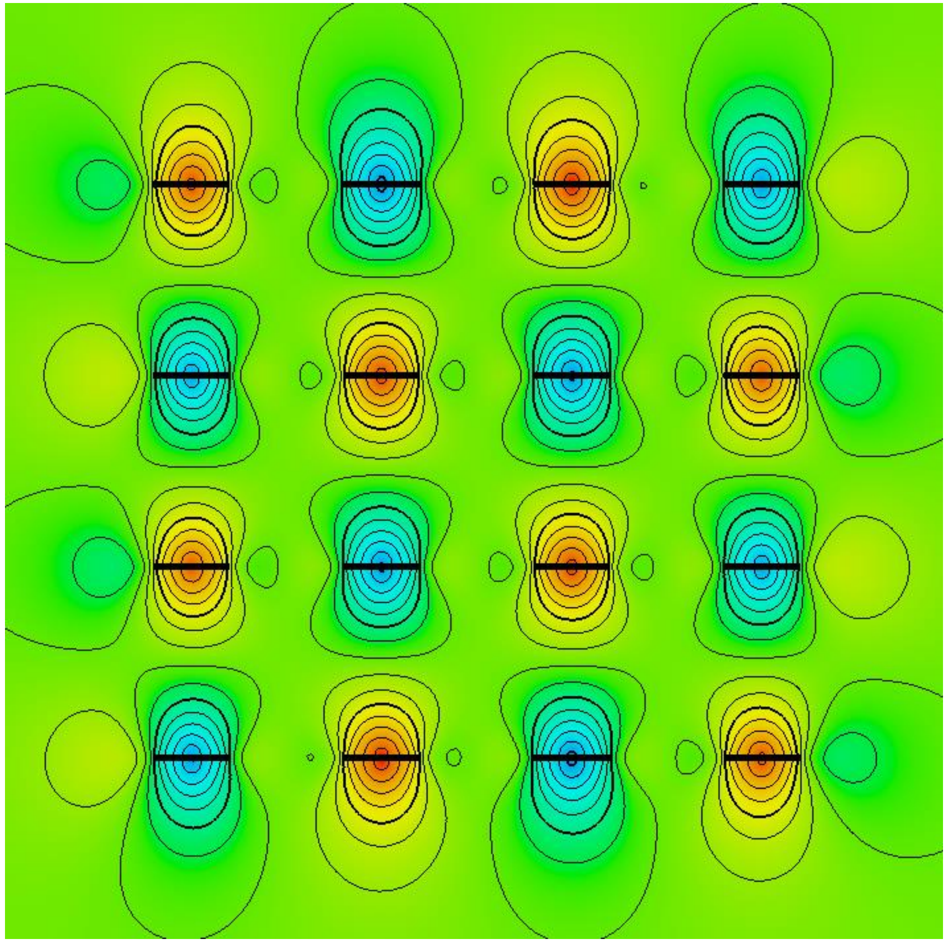
— fracture

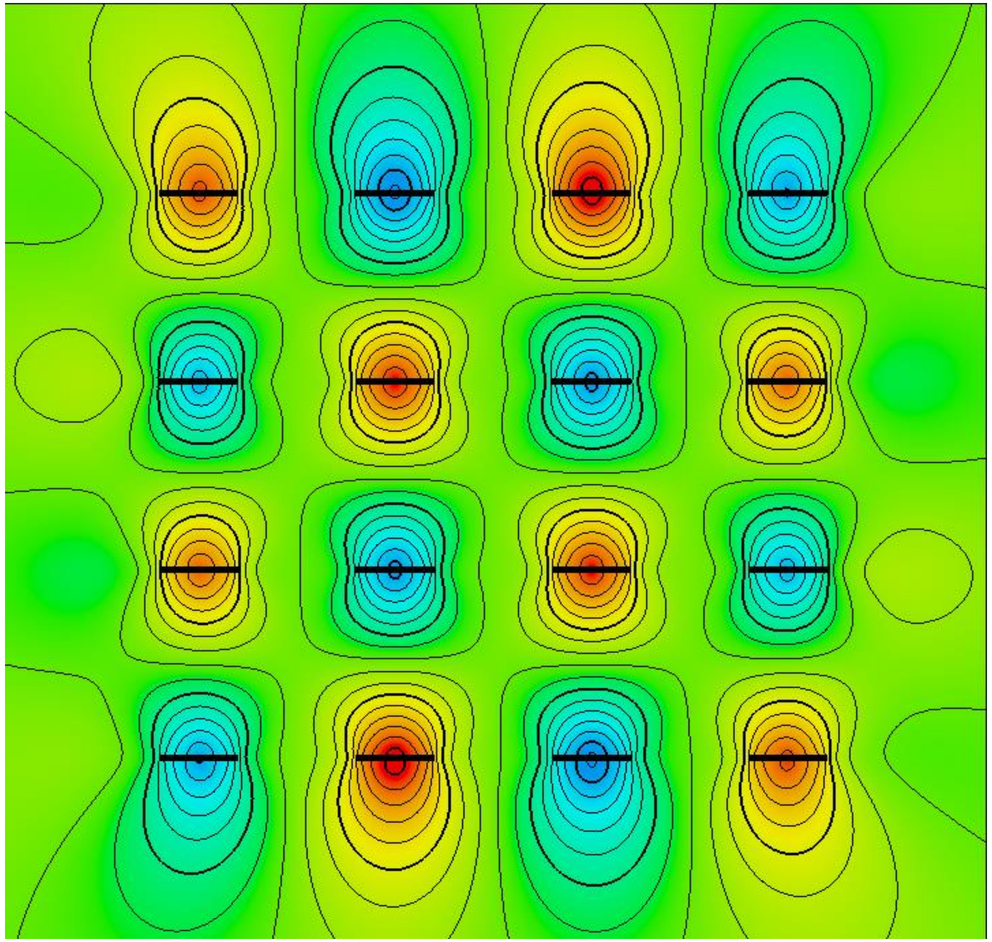
Results of hydrogeomechanical modeling: strains ε_{xx}

Max



Min



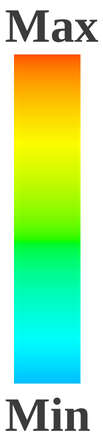


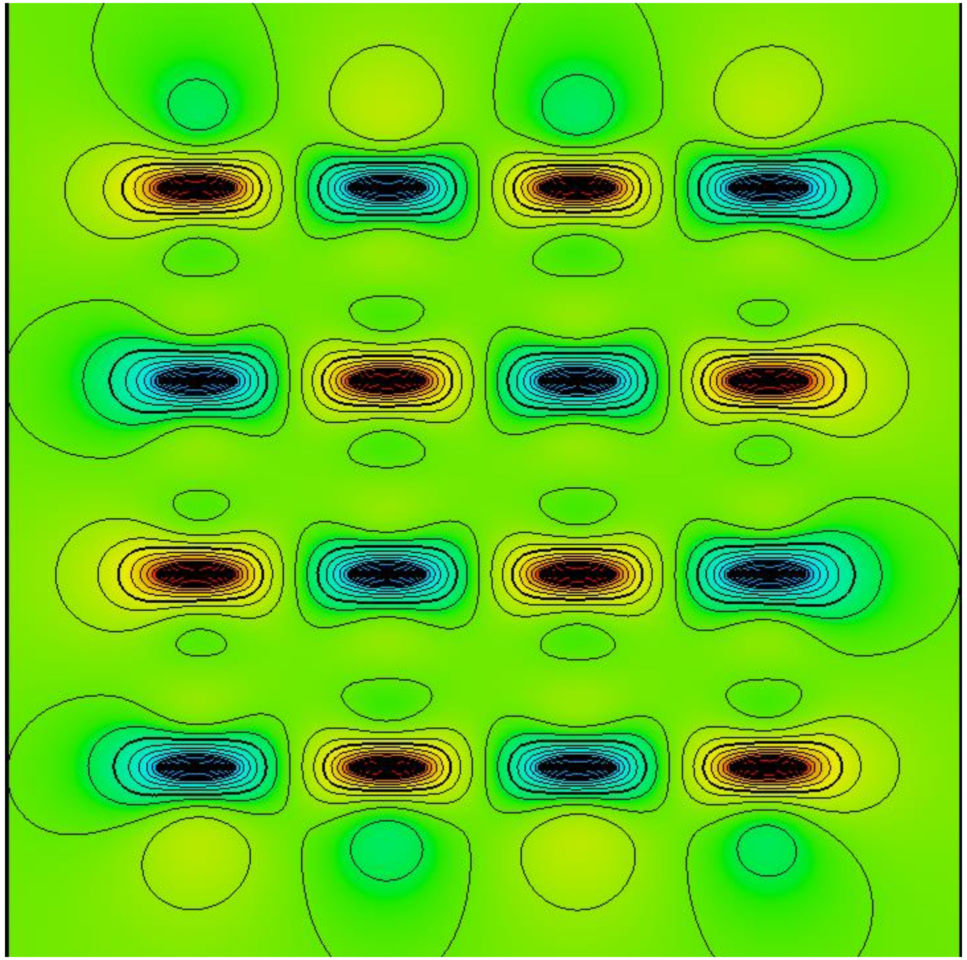
Middle time step

Last time step

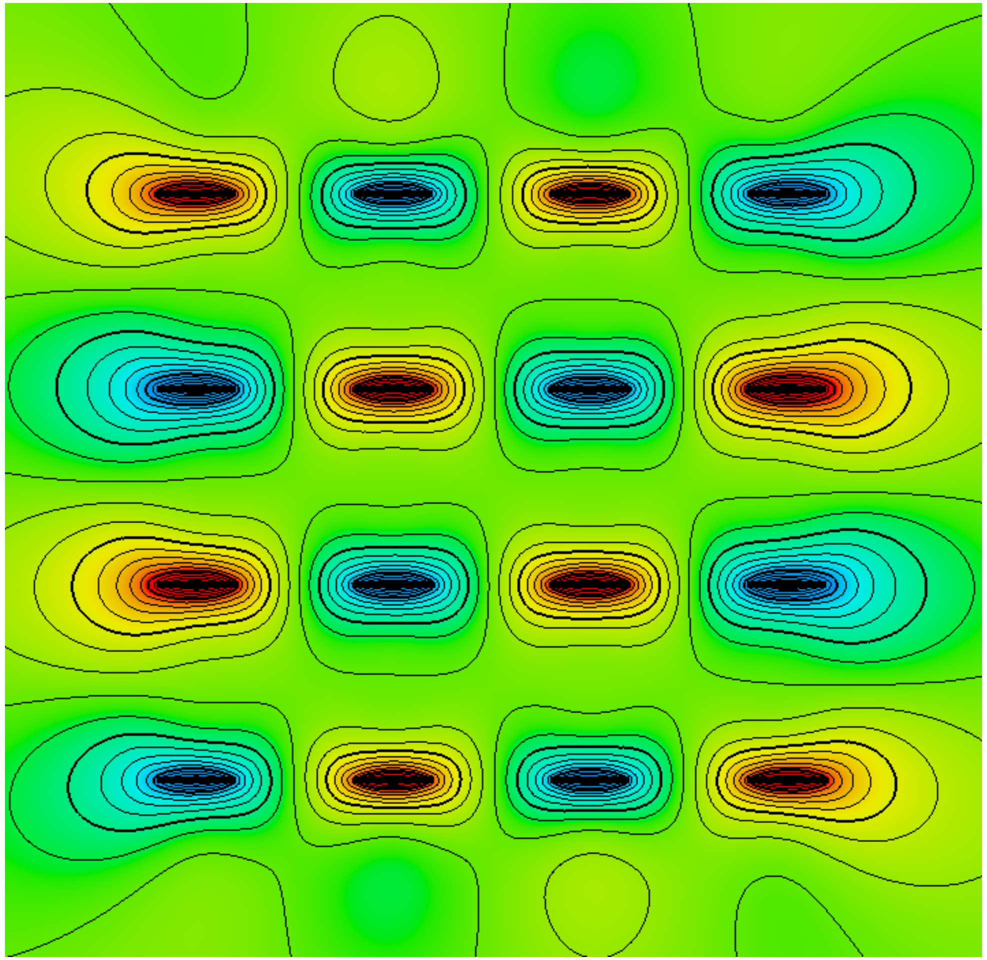
— fracture

Results of hydrogeomechanical modeling: strains ε_{yy}





Middle time step

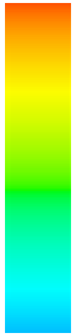


Last time step

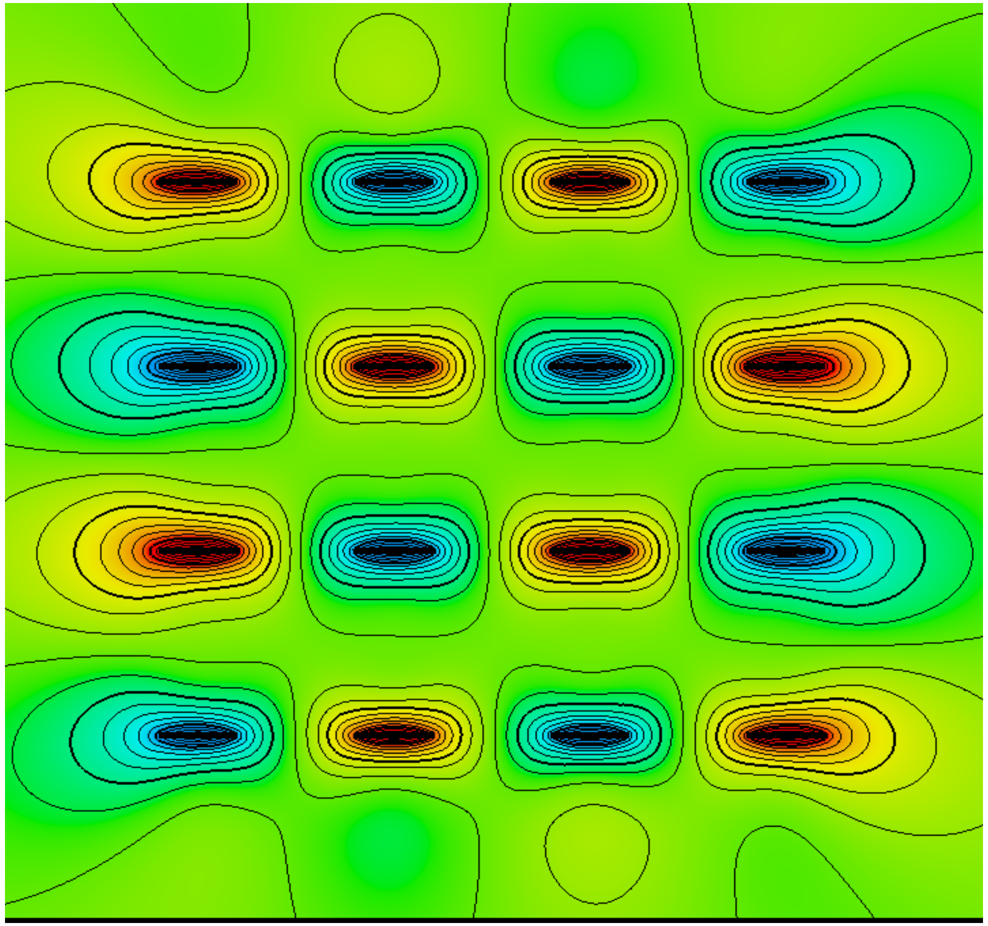
fracture

Results of hydrogeomechanical modeling: displacements u_x

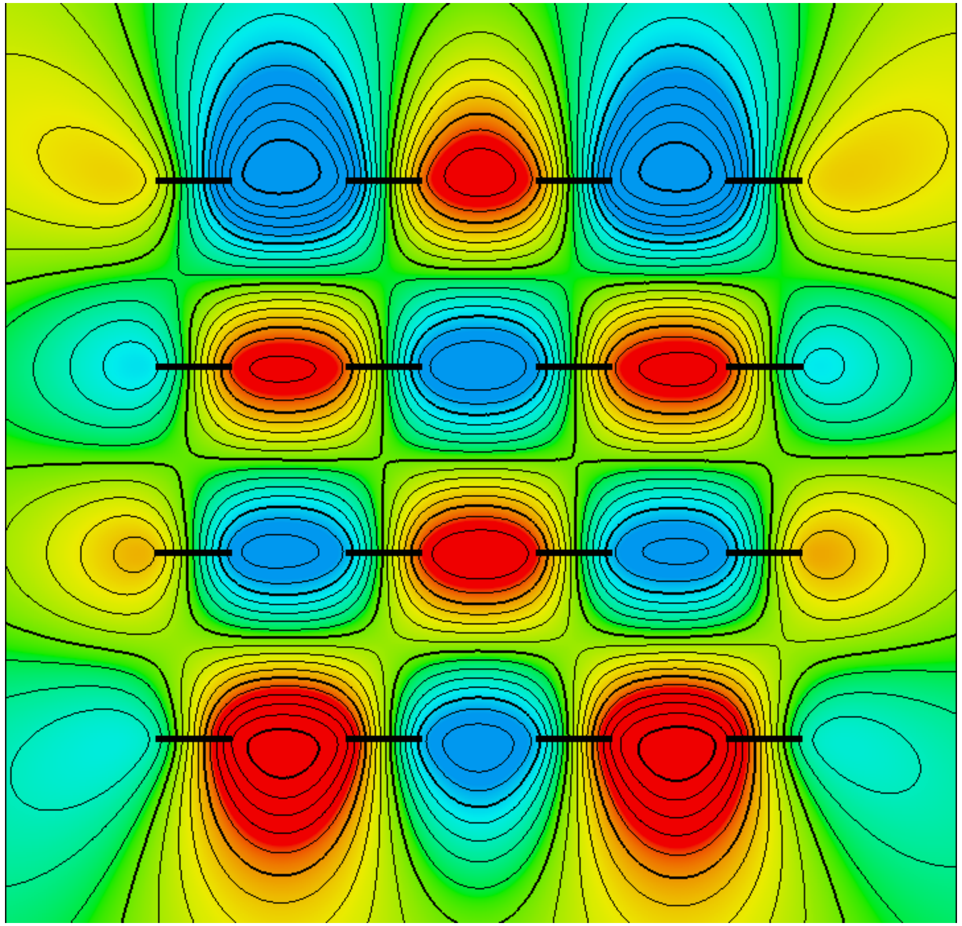
Max



Min



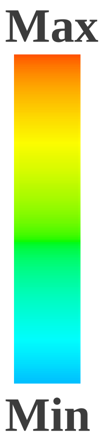
Middle time step

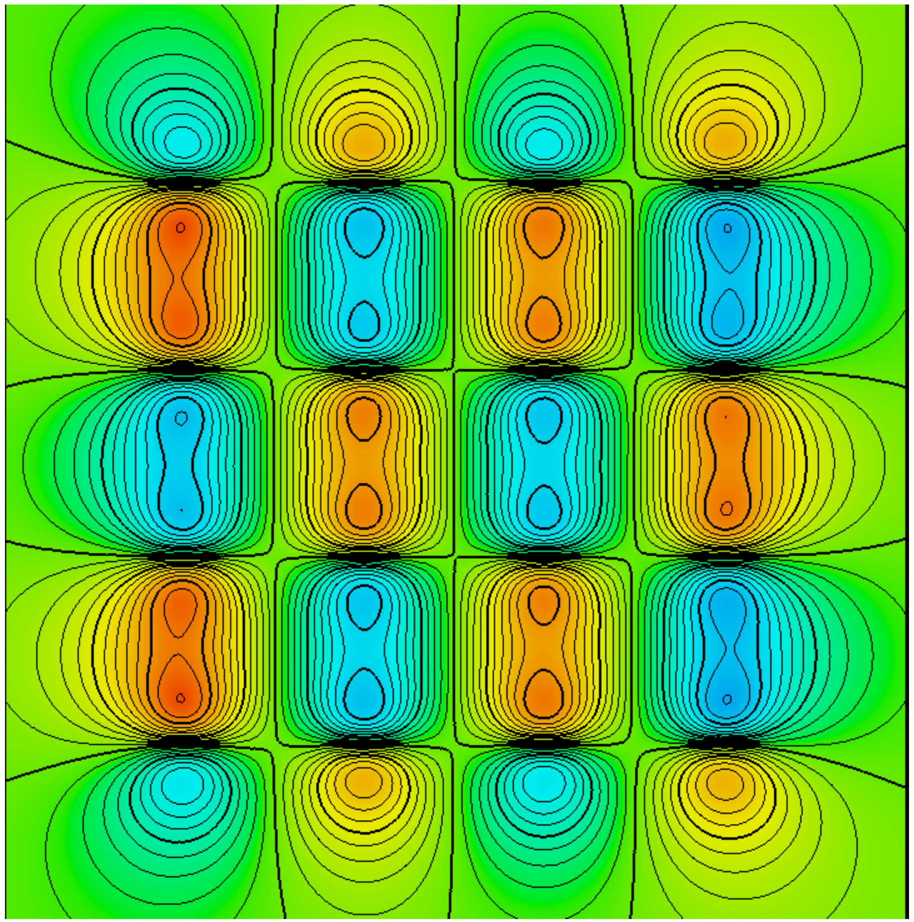


Last time step

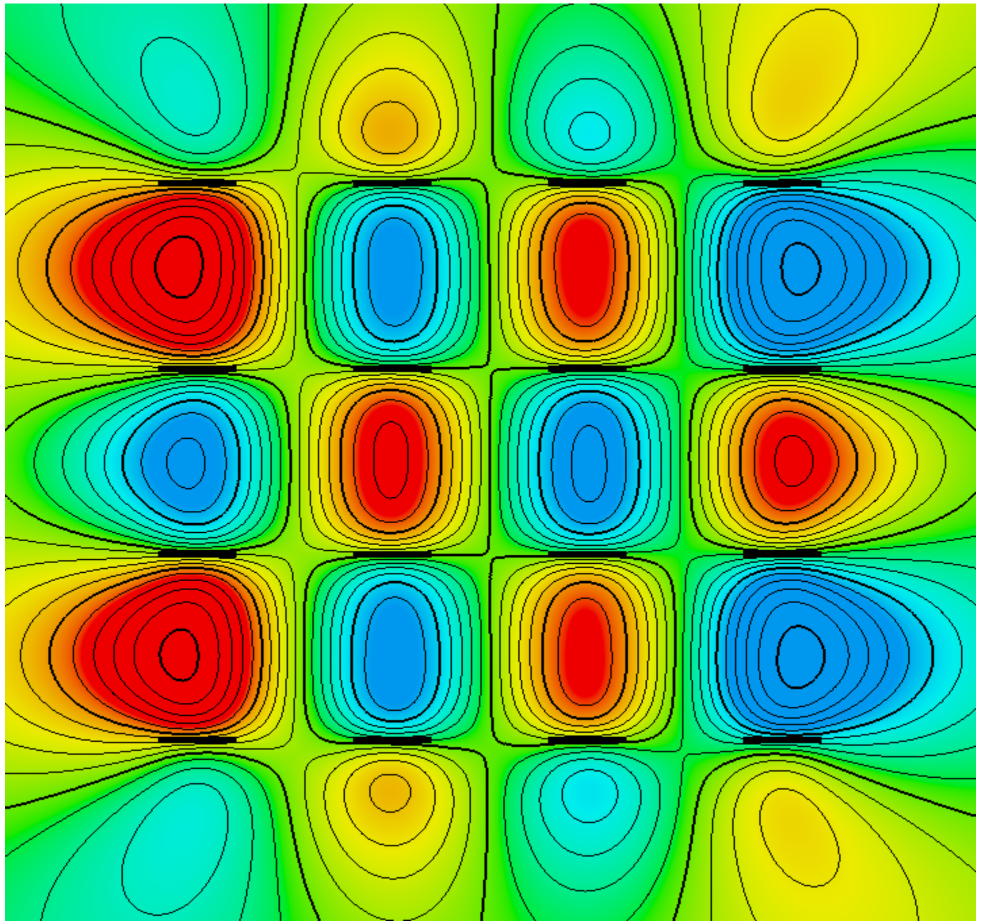
— fracture

Results of hydrogeomechanical modeling: displacements u_y





Middle time step



Last time step

— fracture

Fracture closure pressure: 1D poroelastic horizontal strain model

$$P_f = \sigma_{min} = \frac{\vartheta}{1-\vartheta}\left(\sigma_v - \alpha P_p\right) + \alpha P_p + \frac{vE}{1-v^2}\varepsilon_x + \frac{E}{1-v^2}\varepsilon_y,$$

where P_c – fracture closure pressure;
ϑ – Poisson's ratio;
σ_v – overburden pressure;
α – Biot's poroelastic constant;
Pp – pore pressure;
E-Young's modulus;
$\varepsilon_x, \varepsilon_y$ – tectonic horizontal strains.

However, the use of this analytical equation does not allow to take into account the dynamics of pore pressure and stress state.

Assumptions:

- Uniaxial deformation;
- Does not consider pore pressure distribution;
- Layers are parallel.

Fracture closure pressure: 1D poroelastic horizontal strain model

1D poroelastic horizontal strain model *:*

$$\sigma_h = \frac{\nu}{1-\nu}(\sigma_V - \alpha p_f) + \alpha p_f + \frac{\nu E}{1-\nu^2}\varepsilon_x + \frac{E}{1-\nu^2}\varepsilon_y,$$

Parameters:

$$\nu = 0{,}27$$

$$\alpha = 0{,}8$$

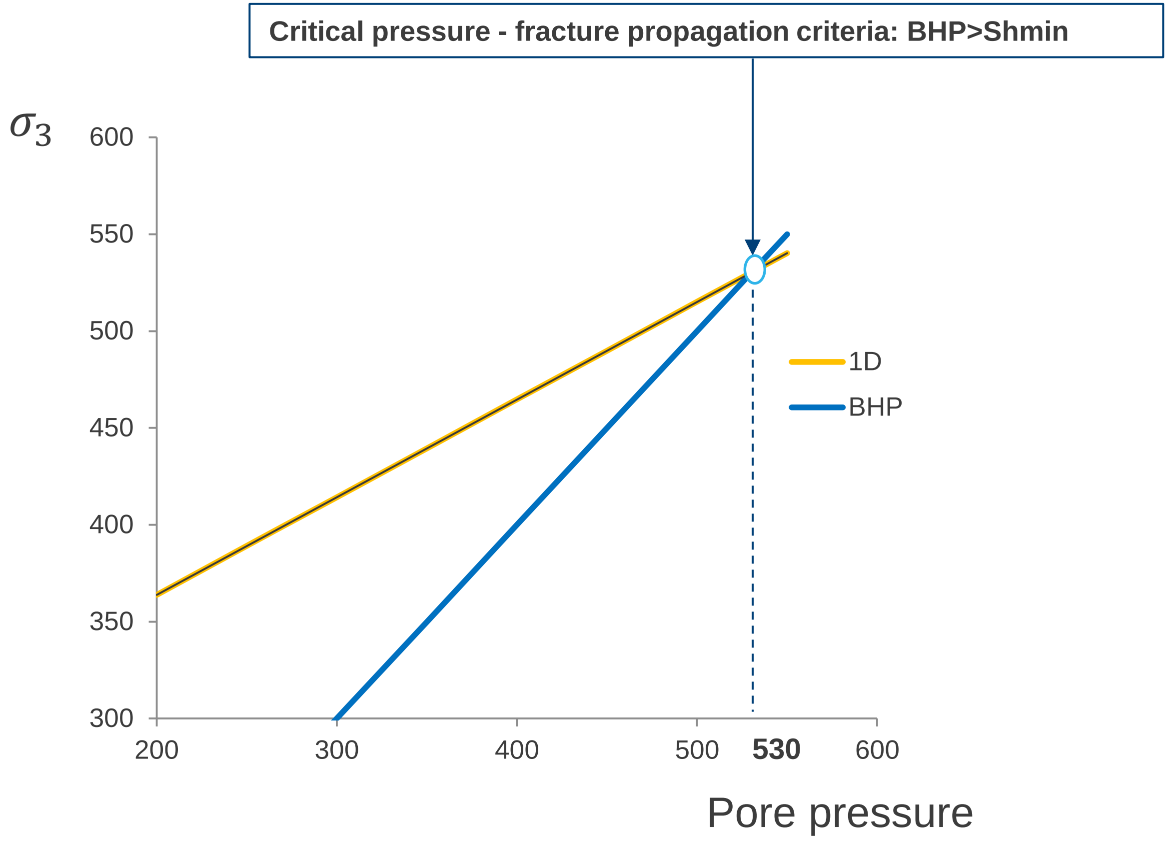
$$E = 19\ \text{GPa}$$

Results:

$$\delta\sigma_{min} = \alpha\frac{1-2\nu}{1-\nu}\delta p_{delta}$$

$$\delta\sigma_{min} = A \cdot \delta p_{delta}$$

$$A = 0.5$$



Fracture closure pressure: comparison of 1D poroelastic horizontal strain model and hydrogeomechanical simulation

1D poroelastic horizontal strain model :

$$\delta\sigma_{min} = \alpha \frac{1 - 2\nu}{1 - \nu} \delta p_{delta}$$

$$A = 0.5$$

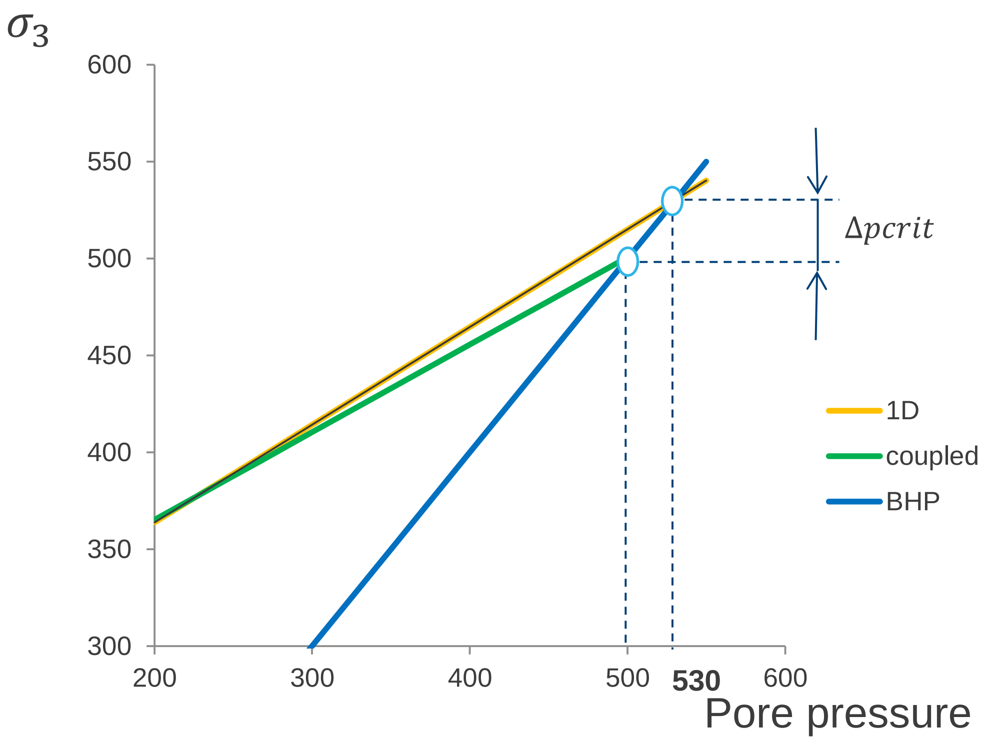
Parameters:

$$a = 500 \text{ m}$$
$$b = 250 \text{ m}$$
$$x_f = 100 \text{ m}$$

Hydrogeomechnical coupling:

$$\delta\sigma_{min} = A \cdot \delta p_{delta},$$

$$A = 0.45$$



! The difference between critical pressure is 30 atm

Development of a semi-analytical model

Conditions:

- **Linear periodic system.**
- **Region under consideration is a rectangular element with 2 injection and 2 production wells with hydraulic fracturing.**
- **Constant pressure p_0 at the boundary of the element.**
- **Bottomhole pressure at the boundary if fracture.**

Algorithm:

I. Hydrodynamical model:

$$\frac{\partial p}{\partial t} + \varkappa \Delta p = 0$$

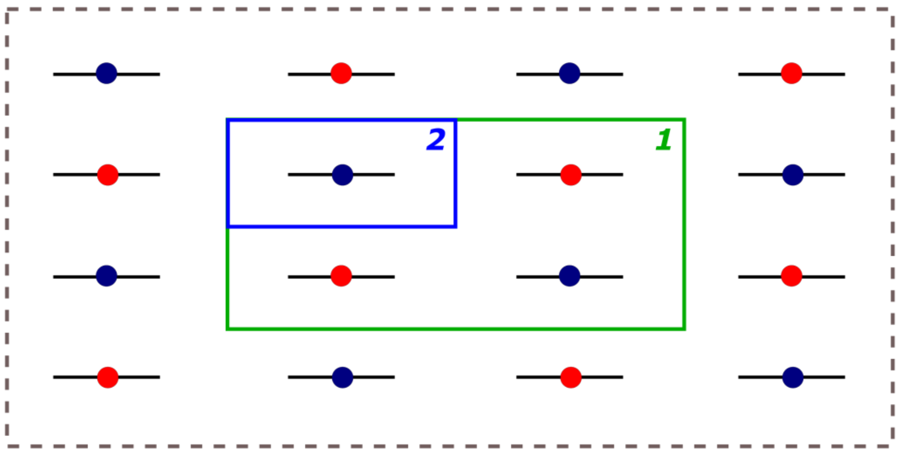
II. Poroelastic model:

$$\partial_j \sigma_{ij} = 0$$

$$\sigma_{ij} = \sigma_{ij}{}^0 + (\lambda\theta + \alpha p_{delta})\delta_{ij} + 2\mu\varepsilon_{ij}$$

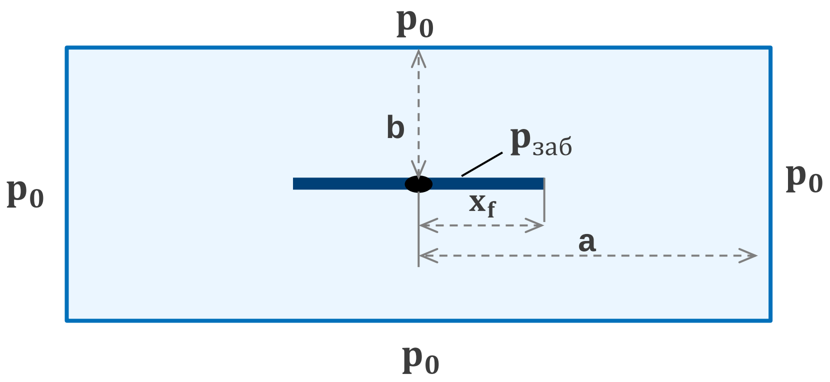
III. Fracture propagation criteria

а)



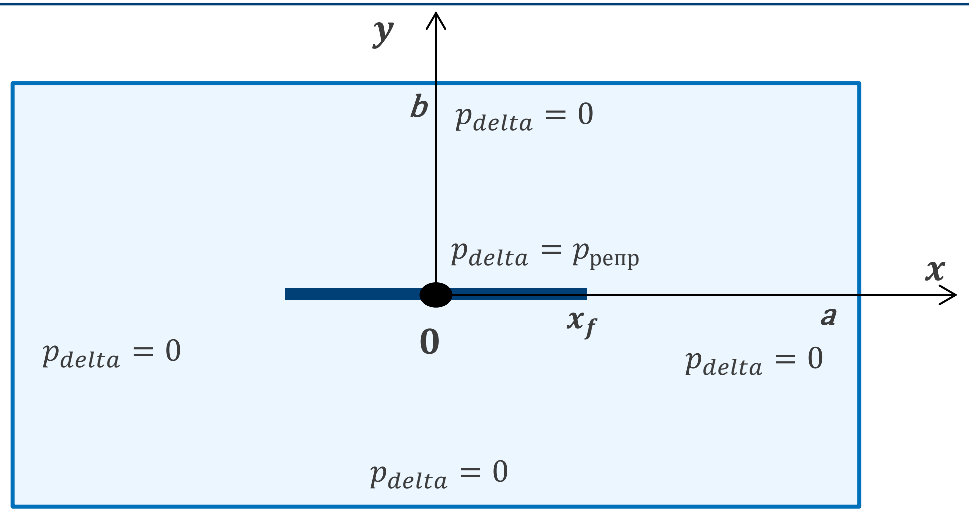
1 –Poroelastic model boundary; 2 – hydrodynamical model boundary

б)



hydrodynamical model boundary.

Hydrodynamical model



- $p_{delta} = p - p_0$
- Uniform pressure in fracture

Mathematical model:

$$\begin{cases} \Delta p_{delta} = 0 \\ p_{delta}|_{трещина} = p_{репр} \\ p_{delta}|_{границы} = 0 \end{cases}$$

$$p_{delta} = p_{репр} \cdot f(x,y)$$

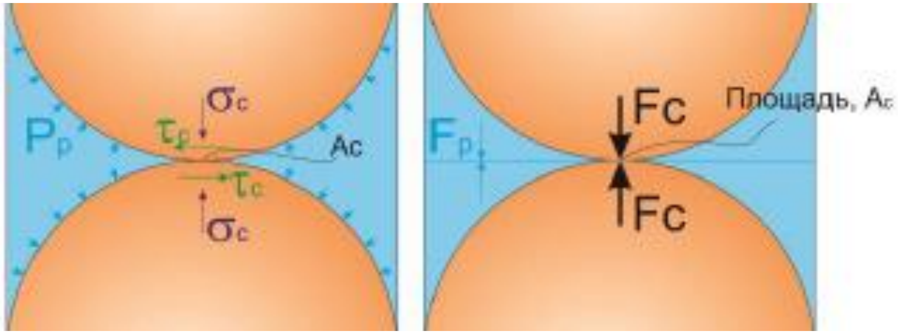
$$\Rightarrow \begin{cases} \Delta f(x,y) = 0 \\ f(|x| < x_f, y) = 1 \\ f(x = \pm a, y) = 0 \\ f(x, y = \pm b) = 0 \end{cases}$$

Method:

Numerical solution of the system: iterative method using finite difference schemes

$$f_{i,j}^{k+1} = \frac{w}{2\left({h_x}^2 + {h_y}^2\right)}\left({h_y}^2 f_{i-1,j}^{k+1} + {h_y}^2 f_{i+1,j}^{k} + {h_x}^2 f_{i,j-1}^{k+1} + {h_x}^2 f_{i,j+1}^{k}\right) + (1-w) f_{i,j}^{k}$$

Poroelastic model



Hooke's law with poroelasticity::

$$\sigma_{ij} = {\sigma_{ij}}^0 + (\lambda\theta + \alpha p_{delta})\delta_{ij} + 2\mu\varepsilon_{ij}$$

$$\sigma_{delta\, ij} = (\lambda\theta + \alpha p_{delta})\delta_{ij} + 2\mu\varepsilon_{ij}$$

Equilibrium equation:

$$\partial_j\sigma_{ij} = 0 \quad \rightarrow \quad \partial_j\sigma_{delta\, ij} = 0$$

$$\sigma_{delta\, zz} = \sigma_{delta\, xz} = \sigma_{delta\, yz} = 0$$

- Periodic boundary conditions on displacements

System of partial differential equations for u, v :

$$\begin{pmatrix} (\lambda A + 2\mu)\frac{\partial^2}{\partial x^2} + \mu\frac{\partial^2}{\partial y^2} & (\lambda A + \mu)\frac{\partial^2}{\partial x\partial y} \\ (\lambda A + \mu)\frac{\partial^2}{\partial x\partial y} & (\lambda A + 2\mu)\frac{\partial^2}{\partial y^2} + \mu\frac{\partial^2}{\partial x^2} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -A\alpha\frac{\partial p}{\partial x} \\ -A\alpha\frac{\partial p}{\partial y} \end{pmatrix}$$

Displacements

$$\boldsymbol{u, v}$$

Fourier transformation:

Direct

$$\hat{f}(k_x, k_y) = \iint f(x,y)e^{-ik_x x}e^{-ik_y y}dxdy$$

Inverse

$$f(x,y) = \frac{1}{2\pi}\iint \hat{f}(k_x, k_y)e^{ik_x x}e^{ik_y y}dk_x dk_y$$

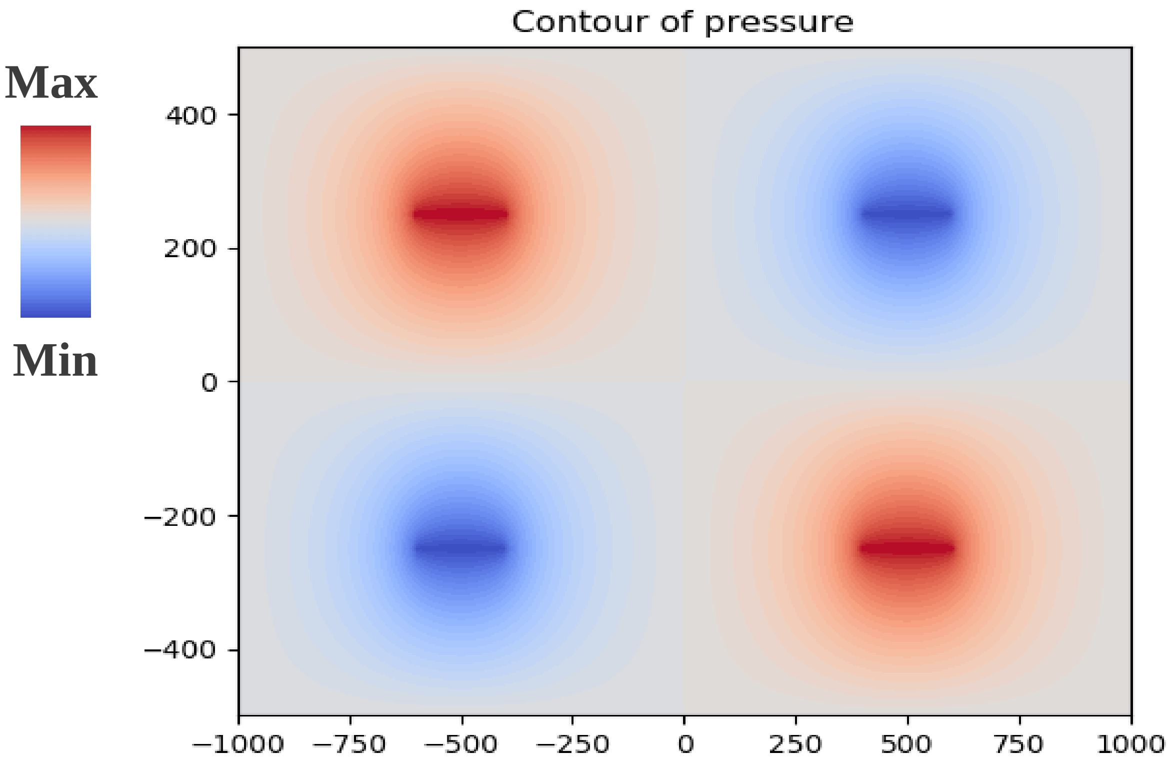
System of linear algebraic equations aor displacements

$$\begin{pmatrix} (\lambda A + 2\mu){k_x}^2 + \mu{k_y}^2 & (\lambda A + \mu)k_x k_y \\ (\lambda A + \mu)k_x k_y & (\lambda A + 2\mu){k_y}^2 + \mu{k_x}^2 \end{pmatrix}\begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} = \begin{pmatrix} ik_x A\alpha\hat{p} \\ ik_y A\alpha\hat{p} \end{pmatrix}$$

Comparison of semianalytical model and hydrogeomechanical coupling

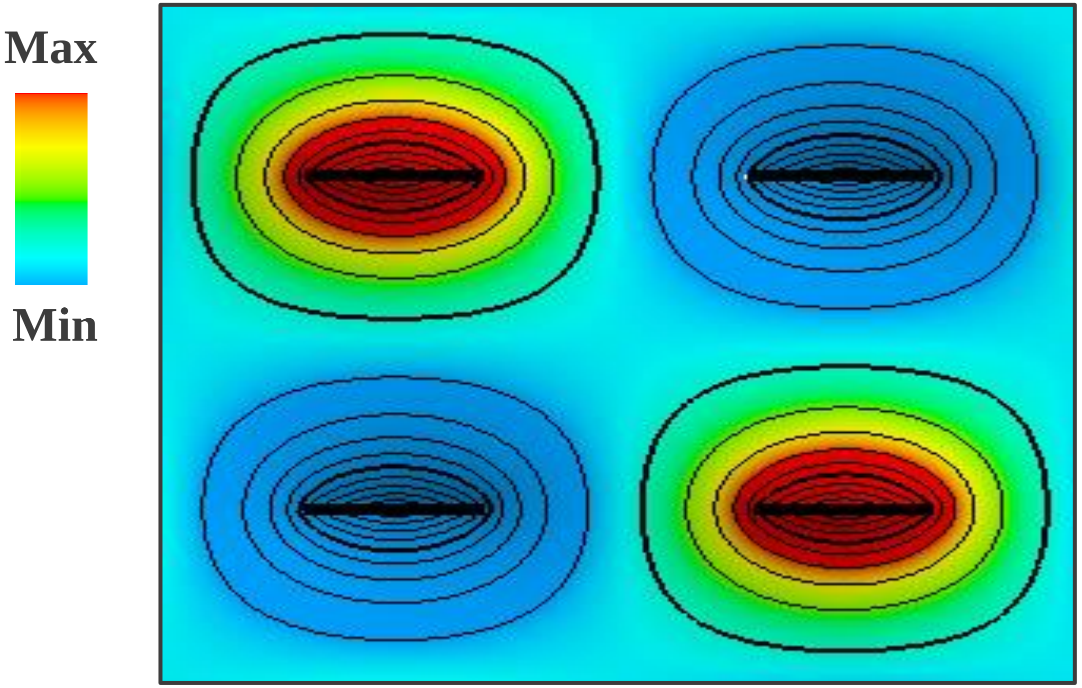
Hydrodynamical model

Semianalytical model



Hydrogeomechanical simulation

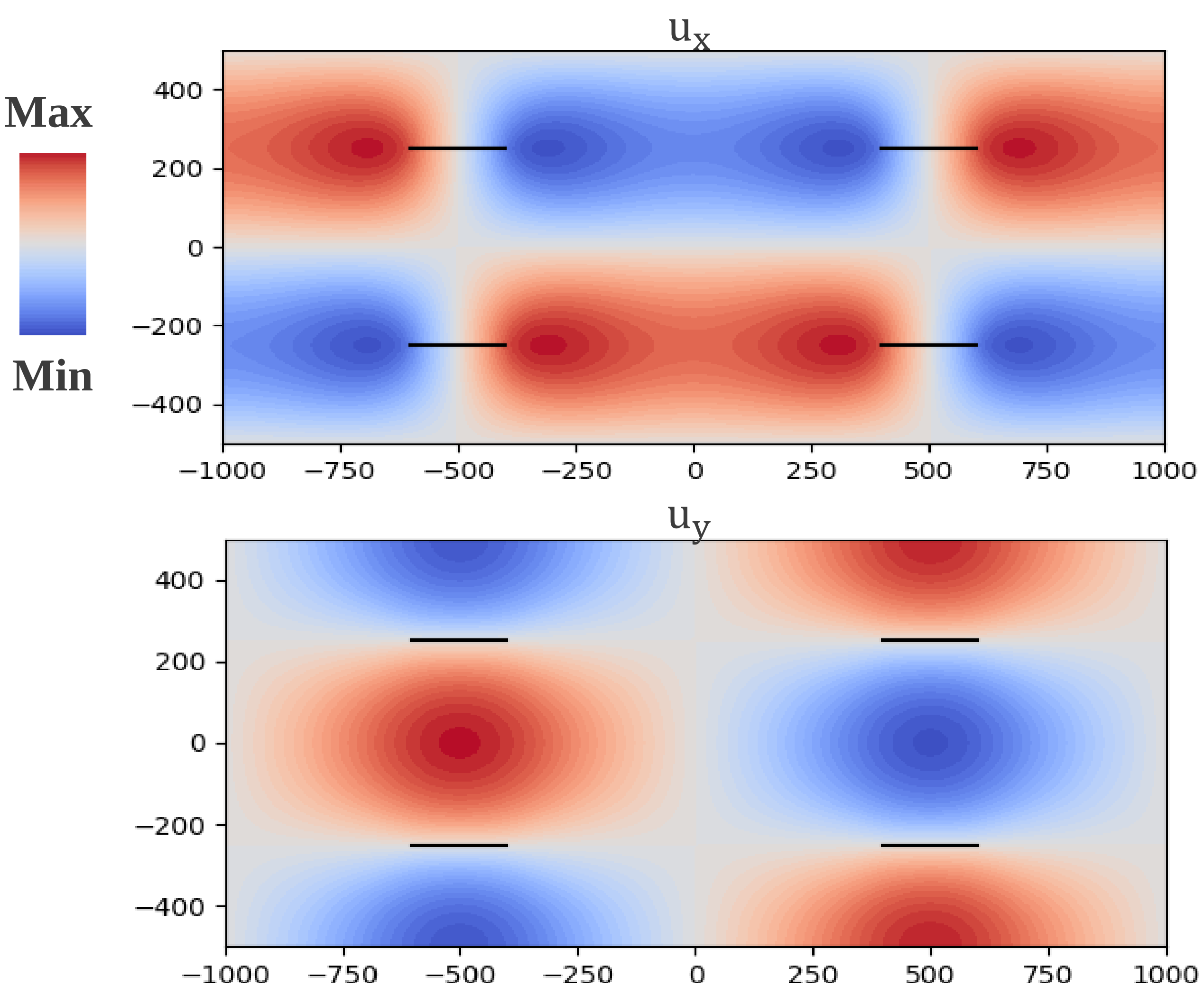
pressure



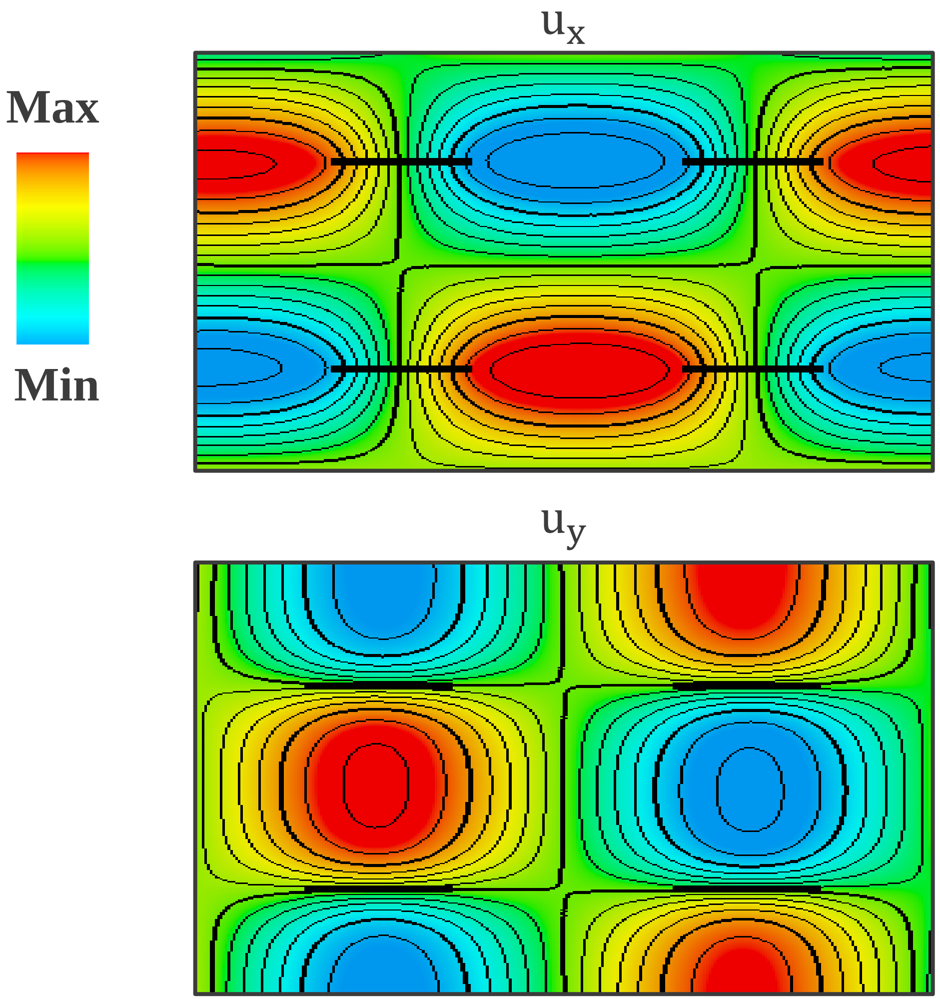
Comparison of semianalytical model and hydrogeomechanical coupling

Displacements

Semianalytical model



Hydrogeomechanical simulation

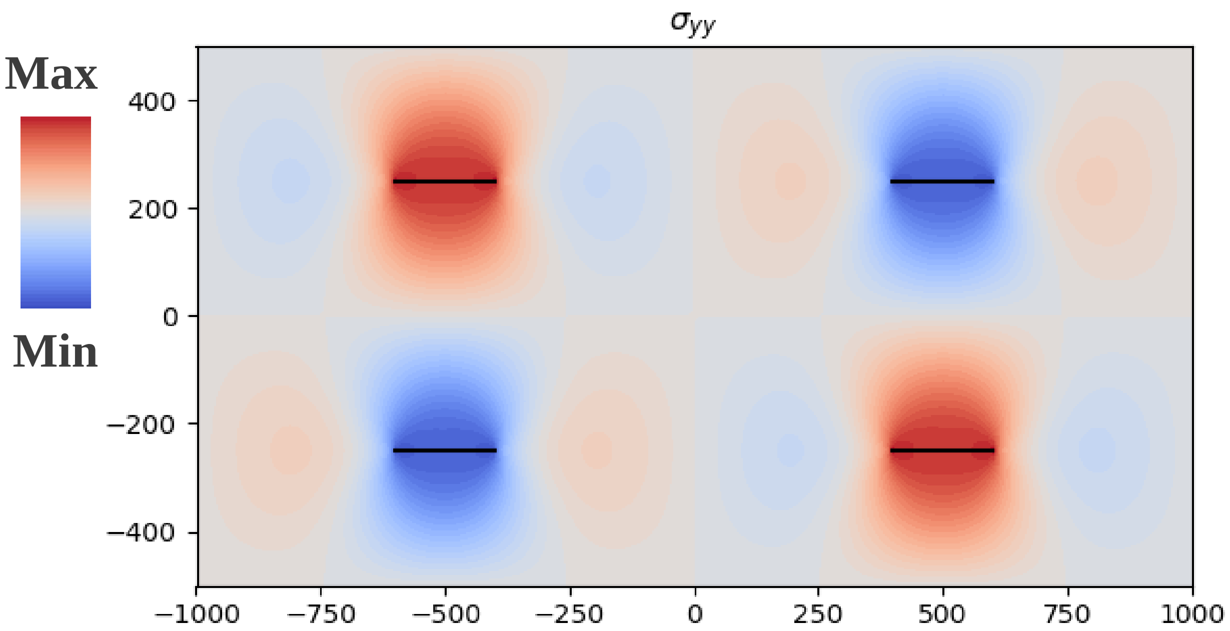


fracture

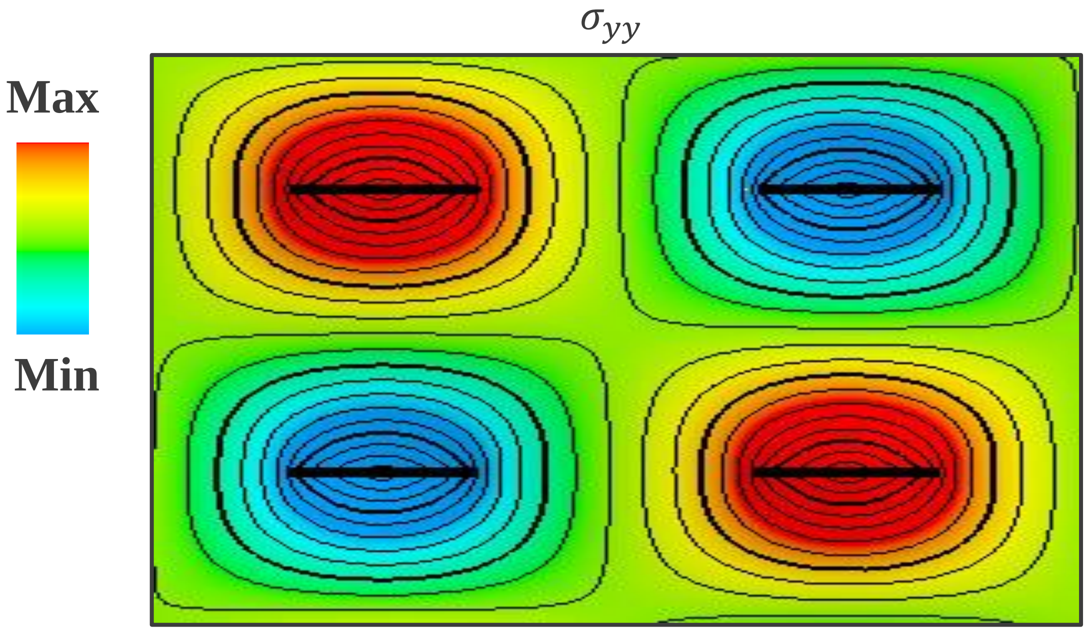
Comparison of semianalytical model and hydrogeomechanical coupling

Stresses

Модель



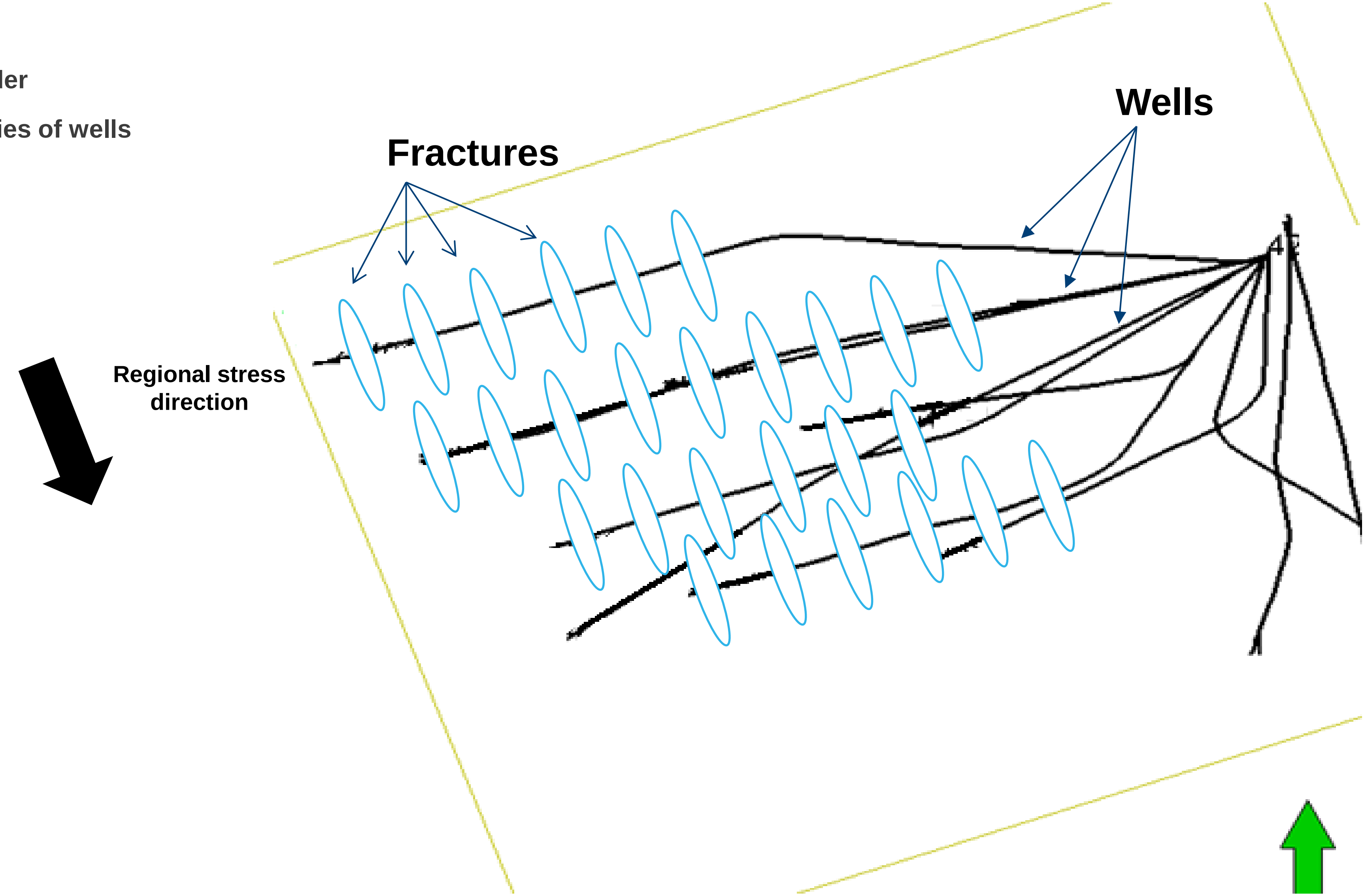
Hydrogeomechanical simulation



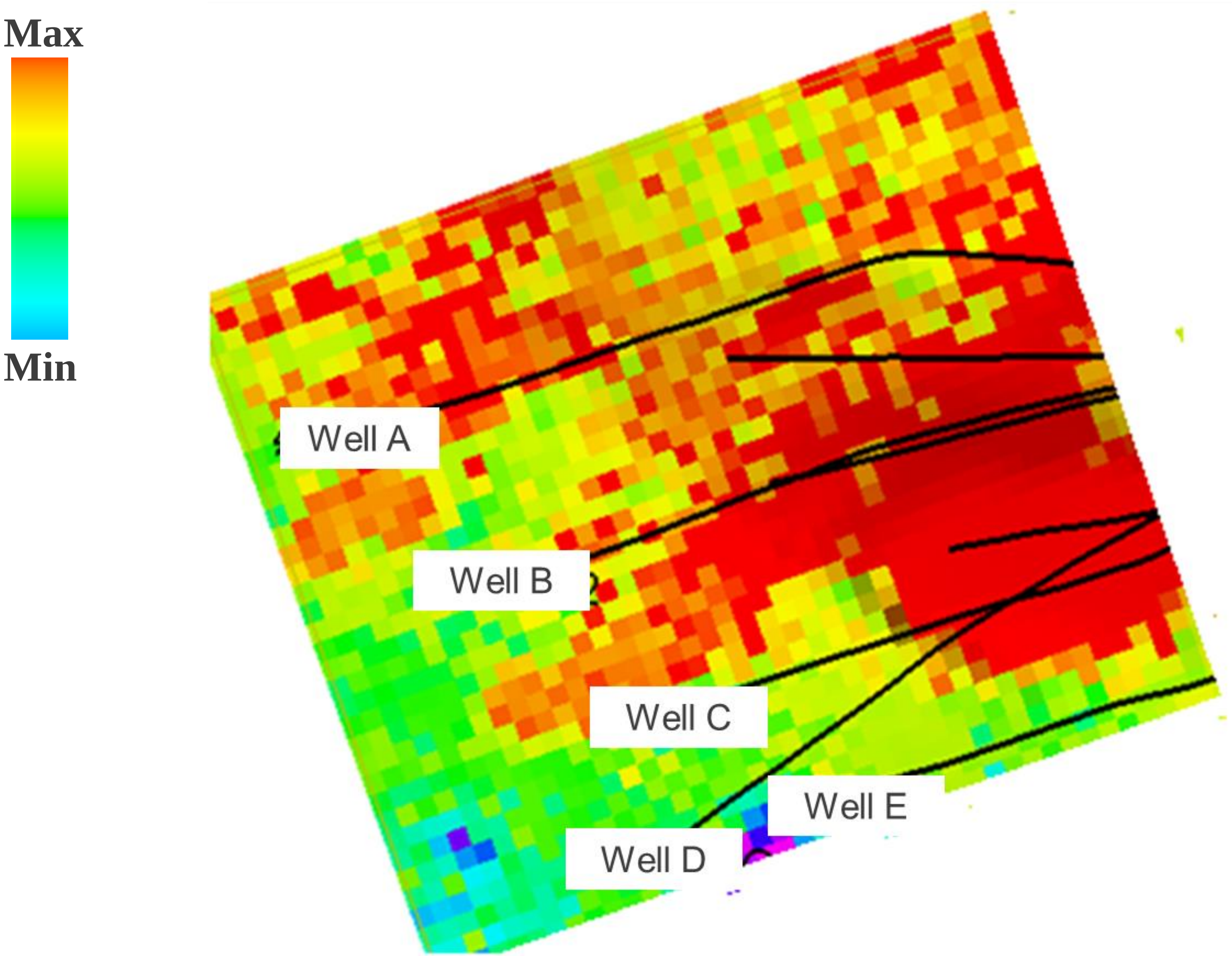
Second case (real case) – no anisotropy, no symmetry

Conditions:

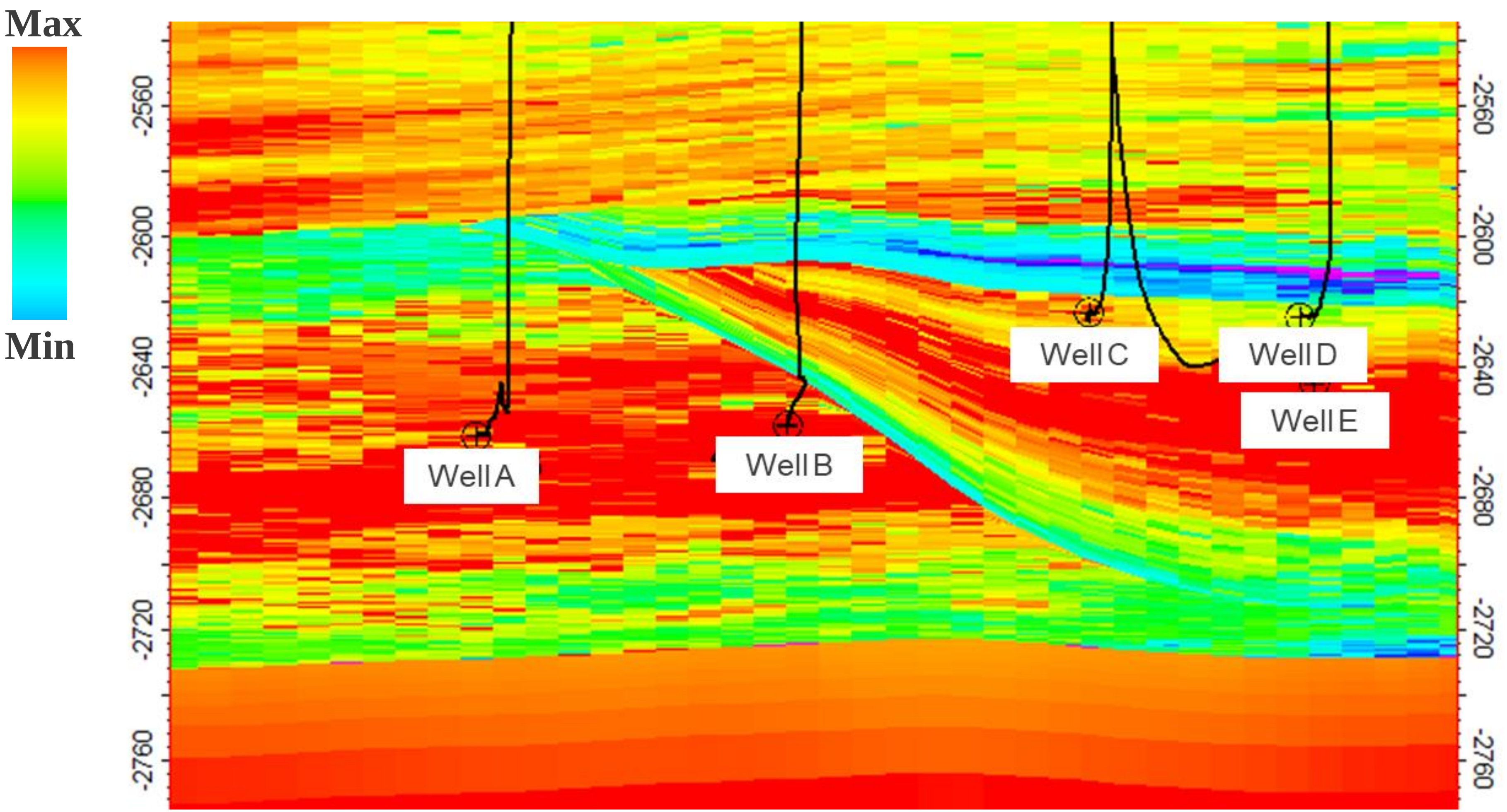
- horizontal wells with fractures in chess order
- fractures are perpendicular to the trajectories of wells
- no anisotropy
- no symmetry



Simulation results: Minimum horizontal stress (view from above)



Simulation results: Minimum horizontal stress (side view)



Conclusions

- **1D horizontal strain model can not be used to predict fracture closure pressure evolution due to the fact that it does not allow to take into account the dynamics of pore pressure distribution**
- **Semi-analytical models can be used in the case of isotropy or weak anisotropy**
- **In case of significant anisotropy it is necessary to use three-dimensional coupled hydrogeomechanical modeling.**

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