The coupling of an enhanced pseudo-3D model for hydraulic fracturing with a model of proppant transport

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HF models

1. 1D models
    - KGD
    - Radial
    - PKN
2. P3D model (1D)
3. Planar model (2D)
4. Full 3D
5. ...

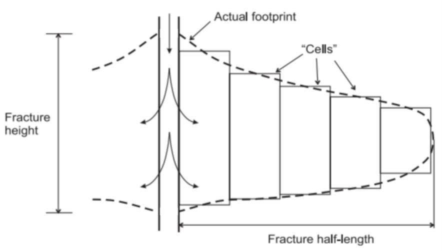


Figure 1: Schematics of P3D fracture geometry [Adachi et al, 2007]

Outline

1. General problem
2. EP3D model (1D)
   - Leak-off
   - Verification
3. Transport model (2D)
   - Num. method
   - Verification
4. Coupling
5. Numerical simulations

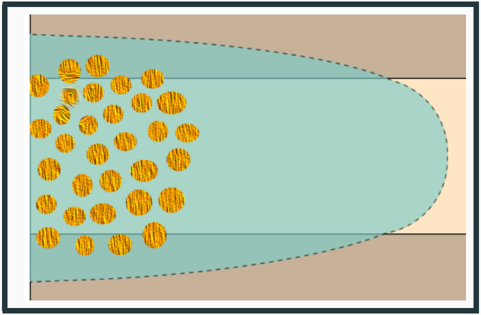


Figure 2: Proppant particles within HF

General Assumptions

1. P3D model (1D) + proppant transport (2D)
2. Linear-elastic 3-layered rock
3. The fluid flow: $q = -\frac{w^3}{12\mu(c_p)}\nabla p$, where $\mu(c_p) = \mu_0 \left(1 - \frac{c_p}{c_{max}}\right)^{-2.5}$
4. Linear-elastic fracture mechanics (LEFM): $K_1 = K_{1C}$
5. Inertial effects are negligible
6. The fluid front coincides with the crack front
7. Leak-off: Carter's law $Q_{leak} = \frac{2C_{leak}}{\sqrt{t - \mathbf{t_0}(x,z)}}$
8. No hydrostatic pressure change
9. The proppant particles are small compared to a characteristic lengthscale
10. No settling

Two-dimensional model

Volume balance

$$\frac{\partial(c_f w)}{\partial t} + \nabla \cdot (c_f w u_f) + Q_{leak} = c_{f,0} Q_0 \delta(x) \psi(z) \quad (1)$$

$$\frac{\partial(c_p w)}{\partial t} + \nabla \cdot (c_p w u_p) = c_{p,0} Q_0 \delta(x) \psi(z) \quad (2)$$

Poiseuille flow and effective viscosity

$$u_f = u_p = -\frac{w^2}{12\mu(c_p)} \nabla p, \quad \mu(c_p) = \mu_0 \left(1 - \frac{c_p}{c_{max}}\right)^{-2.5} \quad (3)$$

Elasticity

$$p(x,z,t) - \sigma_c(x,z) = -\frac{E'}{8\pi} \int_{\Omega(t)} \frac{w(x',z',t)dx'dz'}{\left((x'-x)^2 + (z'-z)^2\right)^{3/2}} \quad (4)$$

Enhanced Pseudo3D model

- Predominance of flow along the x-axis
- The pressure along the Z axis is constant
- Plane strain elasticity in (Y-Z) plane

- Viscous height growth
- Lateral fracture toughness
- Curved tip

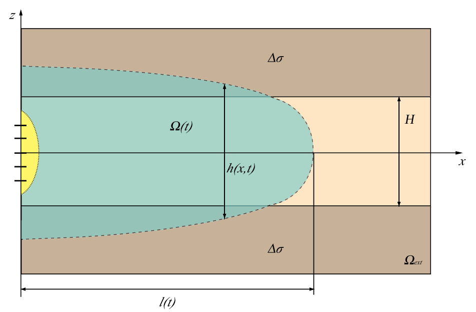


Figure 3: Hydraulic fracture geometry in EP3D model

Enhanced Pseudo3D model

Averaging

$$\bar{w} = \frac{1}{H}\int_{h/2}^{h/2} w dz, \qquad \bar{q} = \frac{1}{H}\int_{h/2}^{h/2} q dz, \tag{5}$$

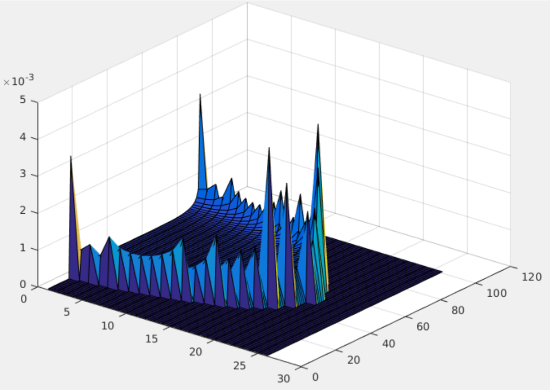
$$Q_0 = \frac{1}{H}\int_{h/2}^{h/2} Q dz, \quad Q_{leak,x} = \frac{1}{H}\int_{h/2}^{h/2} Q_{leak} dz \tag{6}$$

$$\underbrace{\frac{\partial \bar{w}}{\partial t}}_{\text{fracture growth}} + \overbrace{\frac{\partial \bar{q}_x}{\partial t}}^{\text{fluxes}} + \underbrace{Q_{leak,x}}_{\text{leak-off into rock}} = \overbrace{\frac{Q_0}{H}\delta(x)}^{\text{injection rate}}$$

$$\bar{w}(s) = \bar{w}_a(s), \qquad s \to 0$$

[*] E.V. Dontsov, A.P. Peirce (2015): An enhanced pseudo-3D model for hydraulic fracturing accounting for viscous height growth, non-local elasticity, and lateral toughness. Eng.Fract.Mech. 142, 116-139

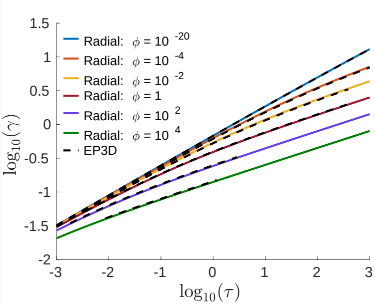
Leak-off

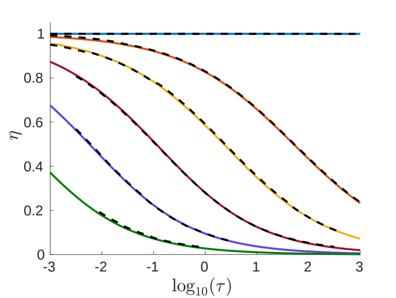


$$Q_{leak} = \frac{2C_{leak}}{\sqrt{t - t_0(x,z)}}, \qquad Q_{leak,x} = \frac{1}{H}\int_{h/2}^{h/2} Q_{leak}\, dz$$

[*] E.D. Carter (1957): Optimum fluid characteristics for fracture extension. Drilling and Production Practices, pp. 261-270

Leak-off: comparison with radial model





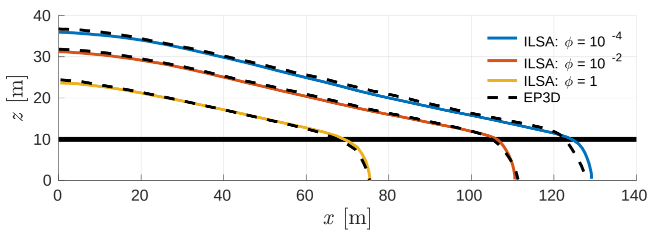
$$L = \left(\frac{Q_0^3 E' t_{mk}^4}{\mu'} \right)^{1/9}, \qquad t_{mk} = \left(\frac{\mu'^5 E'^{13} Q_0^3}{K'^{18}} \right)^{1/2},$$

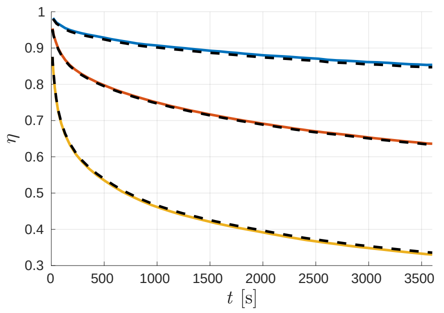
$$\gamma = \frac{R}{L},\ \tau = \frac{t}{t_{mk}},\ \varphi = \frac{\mu'^3 E'^{11} C'^4 Q_0}{K'^{14}}, \qquad \eta = \frac{\text{Volume of the fracture}}{\text{Total injected fluid}}$$

E	ν	Tough. coef. K_{IC}	Inj. rate Q_0	Visc. μ
9.5 GPa	0.2	1 MPa $m^{1/2}$	0.01 m^3/s	0.1 $Pa \cdot s$

[*] E. V. Dontsov (2016) : An approximate solution for a penny-shaped hydraulic fracture that accounts for fracture toughness, fluid viscosity and leak-off.

Leak-off: comparison with ILSA model





[*] E.V. Dontsov, A.P. Peirce (2017): A multiscale implicit level set algorithm (ILSA) to model hydraulic fracture propagation incorporating combined viscous, toughness, and leak-off asymptotics.

Proppant transport

$$\frac{\partial}{\partial x}\left(\frac{w^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{w^3}{12\mu}\frac{\partial p}{\partial z}\right) = Q_{leak} + \frac{\partial w}{\partial t}, \quad V = \begin{bmatrix} u \\ v \end{bmatrix} = -\frac{w(x,z)^3}{12\mu(c_p)}\nabla p,$$

$$\frac{\partial(c_p w)}{\partial t} + \frac{\partial}{\partial x}(c_p w u) + \frac{\partial}{\partial z}(c_p w v) = 0.$$

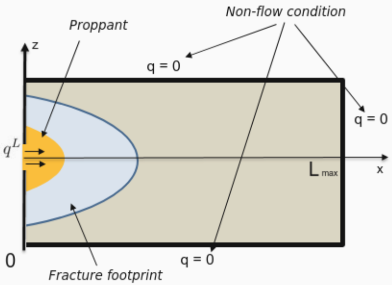


Figure 4: Proppant transport schematics.

Transport scheme

$$F^{LW}_{i-1/2,j} = F^{up}_{i-1/2,j} + \frac{|u_{i-1/2,j}|}{2}\left(1 - \frac{\Delta t}{h_x}|u_{i-1/2,j}|\right)(q_{i,j} - q_{i-1,j})\Phi(\theta_{i,j}).$$

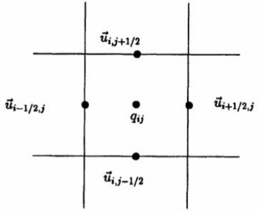
$$\theta_{i,j} = \frac{q_{I,j} - q_{I-1,j}}{q_{i,j} - q_{i-1,j}}, \qquad I = \begin{cases} i, & \text{if } u < 0, \\ i-1, & \text{if } u \geq 0, \end{cases}$$

minmod: $\Phi(\theta_{i,j}) = \max(0, \min(1, \theta_{i,j}))$

superbee: $\Phi(\theta_{i,j}) = \max(0, \min(1, 2\theta_{i,j}), \min(2, \theta_{i,j}))$

van Leer: $\Phi(\theta_{i,j}) = \frac{\theta_{i,j} + |\theta_{i,j}|}{1 + |\theta_{i,j}|}$

MC: $\Phi(\theta_{i,j}) = \max(0, \min((1 + \theta_{i,j})/2, 2, 2\theta_{i,j}))$



Transport: test case

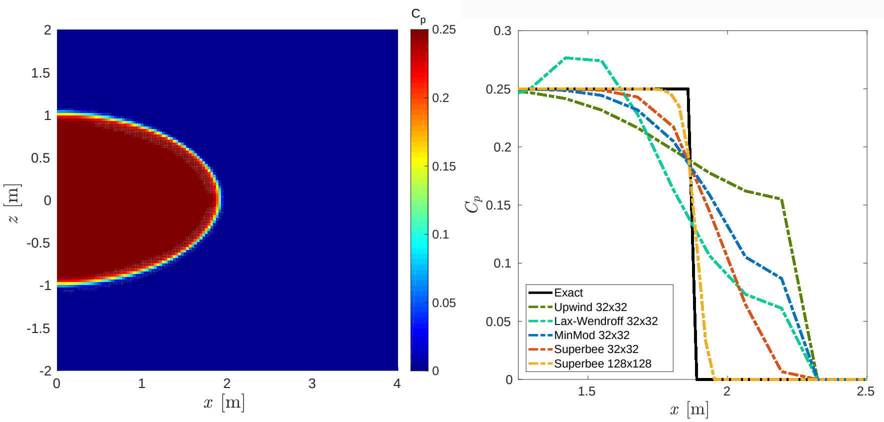


Figure 5: Left figure: numerical solution for the proppant concentration for the test problem using Superbee flux limiter and 128x128 mesh. Right figure: cross section of the solution for proppant concentration at $z = 0$ for different schemes, flux limiters, or meshes.

From 1D to 2D

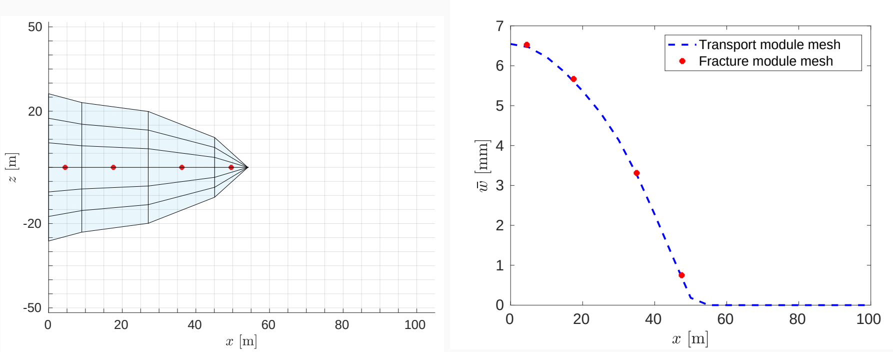
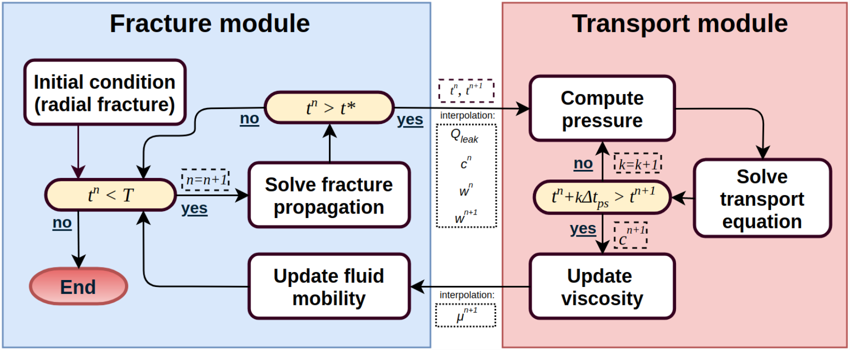


Figure 6: Representation of both numerical meshes for fracture propagation and proppant transport modules. Red dots correspond to the values of effective fracture width $\bar{w}$


Fracture module
Transport module
Initial condition (radial fracture)
$t^n < T$
no
End
yes
n=n+1
Solve fracture propagation
$t^n > t^*$
no
yes
t^n, t^{n+1}
interpolation:
Q_{leak}
c^n
w^n
w^{n+1}
Compute pressure
Solve transport equation
$t^n + k\Delta t_{ps} > t^{n+1}$
no
k=k+1
yes
c^{n+1}
Update viscosity
interpolation:
μ^{n+1}
Update fluid mobility

Parameters

Table 1: Problem parameters for evaluating the effect of schedule on fracture propagation.

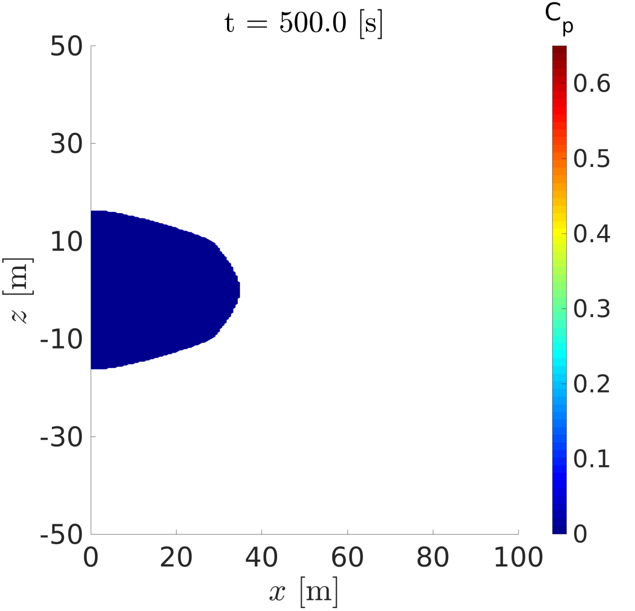
Parameter	Value
K_{1C}	1.0 MPa $\cdot$m$^{1/2}$
$\Delta\sigma$	0.85 MPa
H	20 m
E	9.5 GPa
ν	0.2
μ_f	0.1 Pa·s
Q_0	0.01 m^3·s^{-1}
C'	$1.65 \cdot 10^{-5}$ m·s$^{-1/2}$
t	3600 s

Schedules

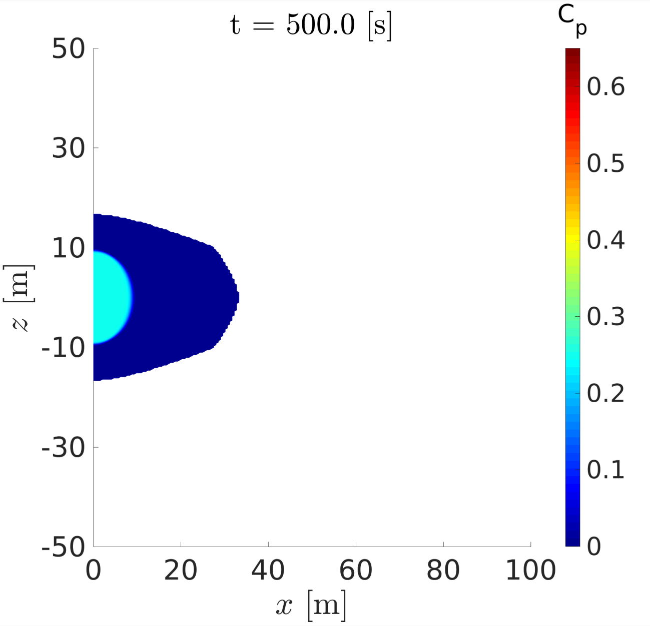
Table 2: Schedule summaries for evaluating the effect of schedule on fracture propagation.

Parameter	Clean	Const	Ramp	Pulses
Q_0	0.01	0.01	0.01	0.01 m^3s^{-1}
Pad	400	400	400	500 s
C_p^{start}	0.0	0.235	0.135	0.275
C_p^{end}	0.0	0.235	0.335	0.295
t_{clean}	-	-	-	100 s
t_{slurry}	-	-	-	400 s

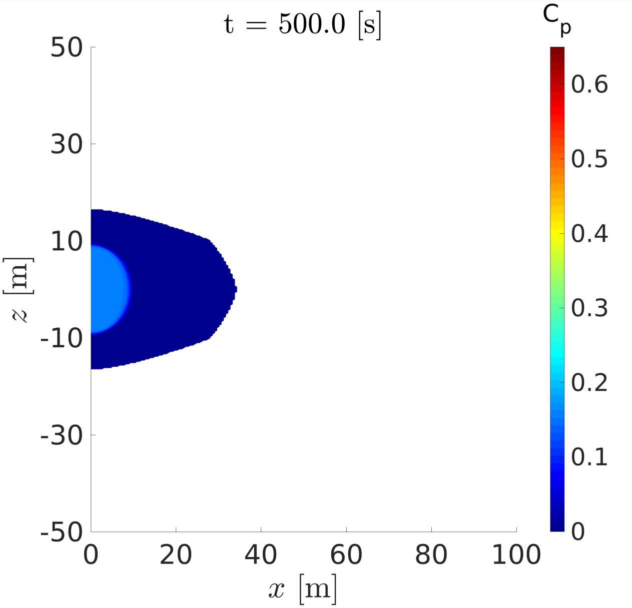
Clean fluid



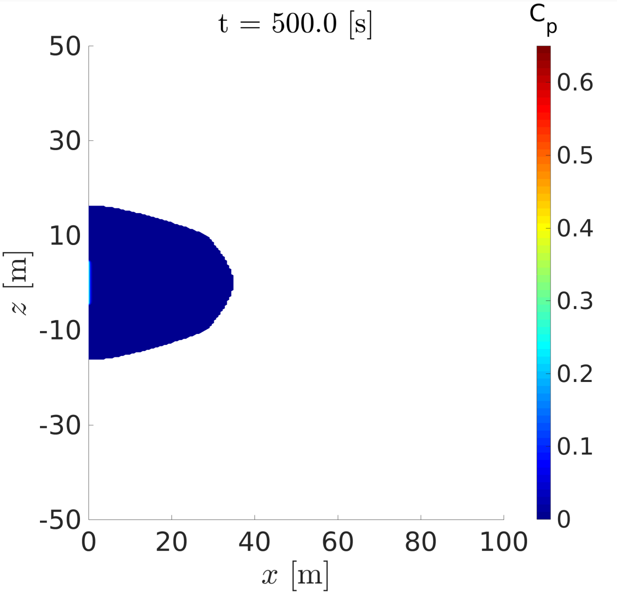
Constant proppant concentration



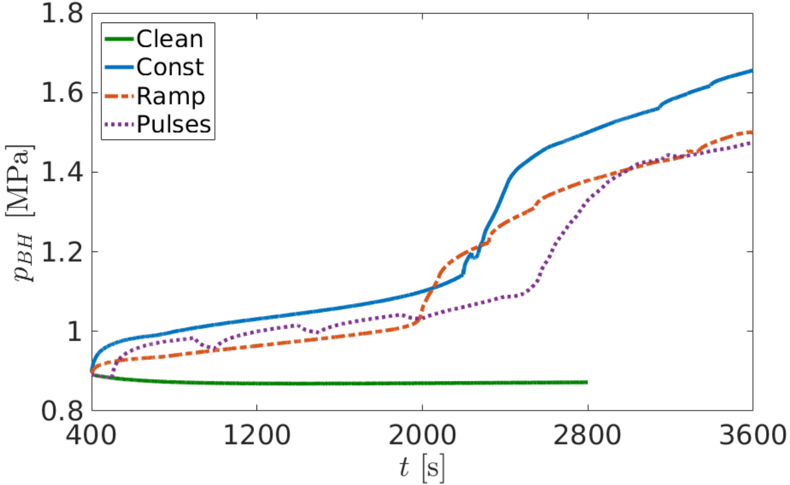
Ramp proppant concentration



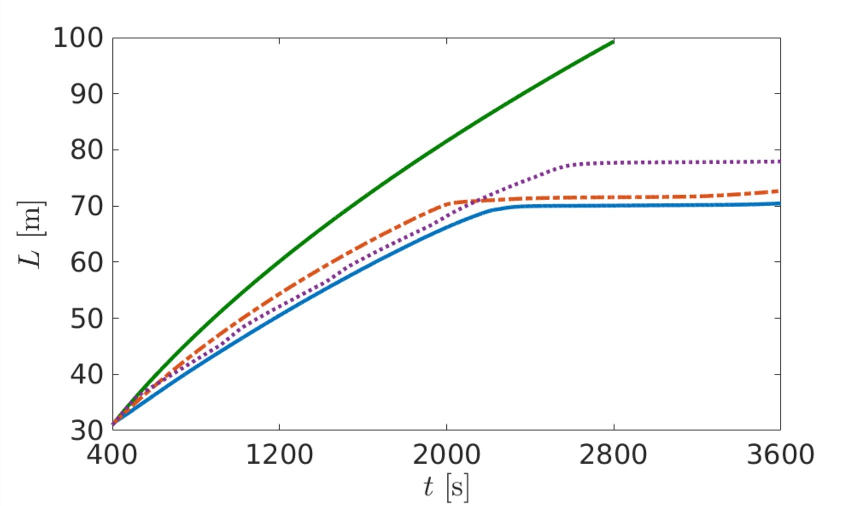
Pulse regime



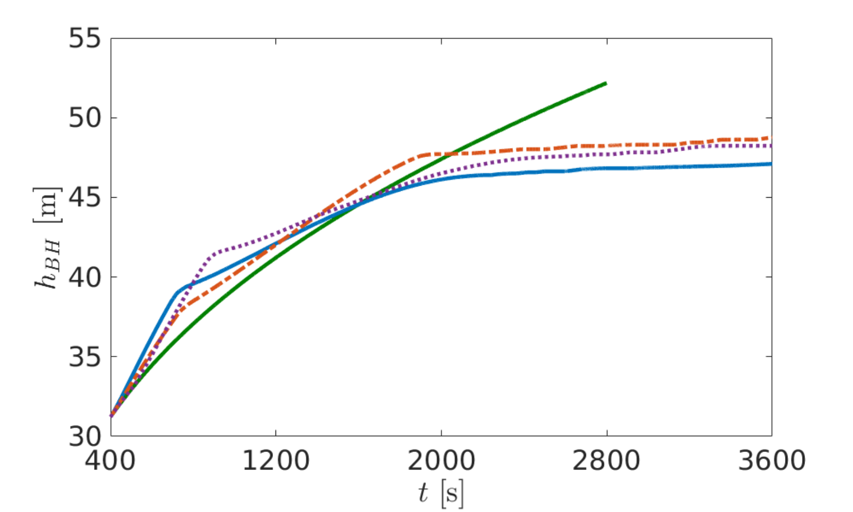
Comparison: pressure



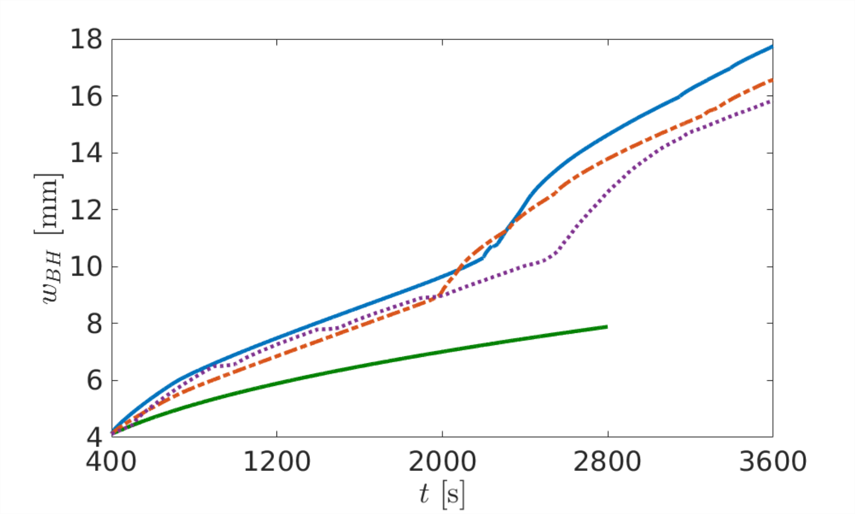
Comparison: length



Comparison: height



Comparison: width



Summary

- EP3D model was supplemented with leak-off. Verification with an analytical solution (radial crack) and two-dimensional ILSA model is carried out;
- Implemented coupling with proppant transport:
  - Transition between one-dimensional and two-dimensional models
  - Implicit integration in EP3D and explicit in transport
  - The consistency of the leaks
- The characteristic simulations of mutual influence of proppant transfer and crack opening are presented.

Thank you for attention!