RHEOLOGY AND MICROSTRUCTURE OF DENSE SUSPENSIONS: applications to the bridging problem

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Shelukhin V.V. Thermodynamics of two-phase granular fluids. Journal of Non-Newtonian Fluid Mechanics. (2018)

Applications:
suspensions, drilling fluids, animal blood, transport of cuttings and proppant

Motivation

Proppant transport in wells and fractures.

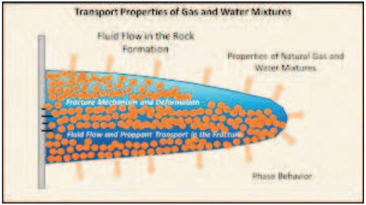


Рис.:

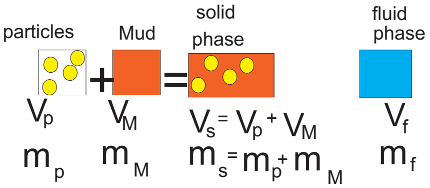
How do

- high particle concentration
- particle-particle interaction
- particle rotation
- non-spherical particle form

impact

- particle's lateral migration
- pressure distribution ?

partial densities and mass concentration

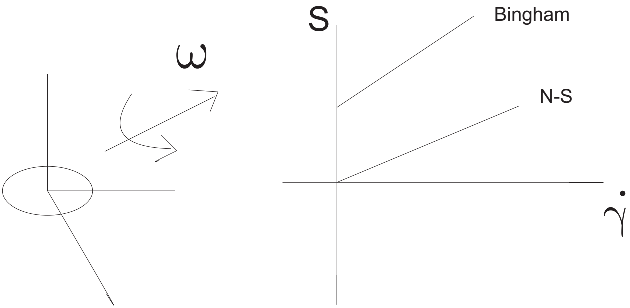


$$\rho_f = \frac{m_f}{m_f + m_p + m_M},\ \rho_s = \frac{m_p + m_M}{m_f + m_p + m_M},\ c = \frac{m_p}{m_f + m_p + m_M},$$

$$\rho_k = \bar{\rho}_k \phi_k, \quad \phi_f + \phi_p + \phi_M = 1$$

Solid phase is a micropolar viscoplastic fluid

Micropolarity and viscoplasticity



(a) - Cosserat E., Cosserat F. Théorie des corps déformable (1909).
(6) - Shvedoff (1890), Bingham (1922).
synergetic effects:

- Shear yield tress and rotational yield stress.
- Two types of plug zone.

V.V. Shelukhin, M. Růžička, On Cosserat-Bingham Fluids, Z. Angew. Math. Mech. (2013).

$$\rho_t + \operatorname{div}\mathbf{j} = 0, \quad \rho_{st} + \operatorname{div}(\rho_s \mathbf{v}_s) = 0, \quad (\rho c)_t + \operatorname{div}(\mathbf{F}_c + \underline{\mathbf{L}}) = 0,$$

$$\mathbf{j}_t + \operatorname{div}(\Pi + \underline{\pi}) = 0, \quad \eta_t + \operatorname{div}\left(\mathbf{F}_\theta + \underline{\mathbf{q}/\theta}\right) = \underline{R/\theta}, \quad \mathbf{s} = J\langle\boldsymbol{\omega}\rangle,$$

$$\mathbf{v}_{2t} + \frac{\partial \mathbf{v}_2}{\partial x}\langle \mathbf{v}_2\rangle = \alpha\nabla\mu + \beta\nabla\theta + \gamma\nabla z + \underline{\mathbf{f}_2}, \quad E_t + \operatorname{div}(\mathbf{Q} + \underline{\mathbf{Q}_1}) = 0,$$

$$(\rho c\mathbf{s})_t + \operatorname{div}(M + \underline{M_1}) = \underline{\mathbf{f}_w}, \quad \mathbf{u} = \mathbf{v}_1 - \mathbf{v}_2,$$ The Gibbs identity:

$$dE_0 = \theta d\eta + \mu d\rho + zd(\rho c) + \underbrace{\mathbf{u}\cdot d\mathbf{j}_0 + \rho c\mathbf{s}d\boldsymbol{\omega}},$$ Galilei' transformation:

$$E = E_0 + \rho\frac{\mathbf{v}_2^2}{2} + \mathbf{v}_2\cdot\mathbf{j}_0, \quad \mathbf{j} = \mathbf{j}_0 + \rho\mathbf{v}_2, \quad \Pi = \Pi_0 + \rho\mathbf{v}_2\otimes\mathbf{v}_2 + \mathbf{v}_2\otimes\mathbf{j}_0 + \mathbf{j}_0\otimes\mathbf{v}_2,$$

$$\mathbf{Q} = \mathbf{Q}_0 + \left(\rho\frac{\mathbf{v}_2^2}{2} + \mathbf{j}_0\cdot\mathbf{v}_2 + E_0\right)\mathbf{v}_2 + \mathbf{j}_0\frac{\mathbf{v}_2^2}{2} + \Pi_0\langle\mathbf{v}_2\rangle, \quad \mathbf{b} \equiv \boldsymbol{\omega} - \operatorname{rot}\mathbf{v}_1/2,$$

Fick's : $\underline{\mathbf{L}} = -\left(\gamma_3\nabla c + \gamma_1\nabla p + \gamma_2\nabla\theta + \gamma_4\nabla\,\mathbf{u}^2 - \alpha_3\operatorname{curl}\boldsymbol{\omega} - \alpha_4\operatorname{curl}\boldsymbol{\omega}\times\mathbf{b}\right)$

Isothermal flows (without viscoplasticity effects)

$$\frac{\partial(\rho_1 \mathbf{v}_1)}{\partial t} + \operatorname{div}(\rho_1 \mathbf{v}_1 \otimes \mathbf{v}_1) = -\frac{\rho_1}{\rho}\nabla p - k\mathbf{u} + \operatorname{div} S_1 - \frac{\rho_1\rho_2}{2\rho}\nabla \mathbf{u}^2 + \rho_1 \mathbf{g},$$

$$\frac{\partial(\rho_2 \mathbf{v}_2)}{\partial t} + \operatorname{div}(\rho_2 \mathbf{v}_2 \otimes \mathbf{v}_2) = -\frac{\rho_2}{\rho}\nabla p + k\mathbf{u} + \operatorname{div} S_2 + \frac{\rho_1\rho_2}{2\rho}\nabla \mathbf{u}^2 + \rho_2 \mathbf{g},$$

$$S_2 = 2\mu(\nabla \mathbf{v}_2)_s + 2\lambda \operatorname{tr}\nabla \mathbf{v}_2 \cdot I, \quad S_1 = 2\eta_s B_s + 2\boxed{\eta_a} B_a$$

$$\frac{\partial(\rho c)}{\partial t} + \operatorname{div}(c\mathbf{j} + \mathbf{L}) = 0, \quad \mathbf{j} = \rho_1 \mathbf{v}_1 + \rho_2 \mathbf{v}_2, \quad \mathbf{u} \equiv \mathbf{v}_1 - \mathbf{v}_2,$$

$$\frac{\partial(\rho c J\langle \boldsymbol{\omega}\rangle)}{\partial t} + \operatorname{div}[(J\langle \boldsymbol{\omega}\rangle \otimes (c\mathbf{j} + \mathbf{L})] = \operatorname{div} N - \epsilon : S_1, \quad B = \nabla \mathbf{v}_1 - \epsilon : \boldsymbol{\omega}$$

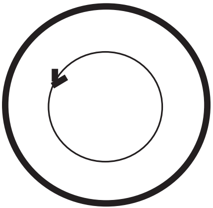
$$N = 2\beta_s A_s + 2\beta_c \operatorname{tr} A \cdot I + 2\beta_a A_a + \alpha_3 \epsilon : \mathbf{L} - \alpha_4 \epsilon : (\mathbf{L} \times \mathbf{b}), \quad A \equiv \nabla \boldsymbol{\omega},$$

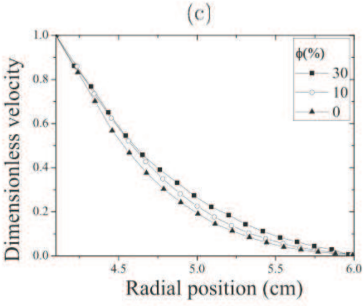
$$\rho_{1t} + \operatorname{div}(\rho_1 \mathbf{v}_1) = 0, \quad \rho_{2t} + \operatorname{div}(\rho_2 \mathbf{v}_2) = 0, \quad \mathbf{b} \equiv \boldsymbol{\omega} - \operatorname{rot} \mathbf{v}_1/2,$$

Skew-symmetric viscosity $\boxed{\eta_a}$: one-phase flows of micropolar fluids

Newtonian fluids : $S = 2\eta(\phi)(\nabla \mathbf{v})_s$, $\eta(\phi)/\eta(0) \overset{*}{=} 1 + E\phi$, $E = 2,5$.

Micrropolar fluids : Theorem $\eta_a/\eta_s = E\phi$, $\phi \sim 0$, $*$Einstein, 1906,





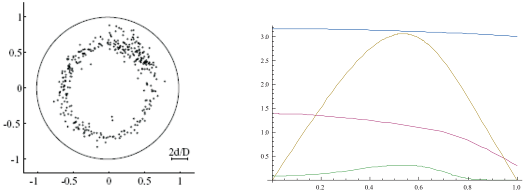
Puc.: Ovarlez et al. 2015

$\eta(\phi)/\eta(0) \overset{**}{=} (1-\phi)^{-E}$, $**$Krieger-Dougherty, 1959. Comparison with experiments $\eta_a/\eta_s = (1-\phi)^{-E} - 1$, $\phi > 0$

Conclusions: **Effective viscosity increases due to particles rotation**

1-st benchmark test: Serge-Silberberg's effect (Nature, 1961):

two-phase granular flows of micropolar fluids



Puc.: (a) Experiment $r_c \approx 0.6r$. (b) Profiles in a channel flow

Nonlinear Fick's law

$$\underline{\mathbf{L}} = -\gamma_3 \nabla c - \gamma_1 \nabla p - \gamma_2 \nabla \theta - \gamma_4 \nabla \left(\mathbf{v}_1 - \mathbf{v}_2\right)^2 +$$

$$+\alpha_3 \operatorname{curl} \boldsymbol{\omega} + \underline{\alpha_4 \operatorname{curl} \boldsymbol{\omega} \times (\boldsymbol{\omega} - \operatorname{rot} \frac{\mathbf{v}_1}{2})}$$

2-nd benchmark test. Two-phase granular fluids. Boycott's effect (Nature, 1920)

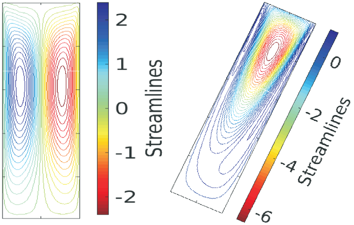


Рис.: Sedimentation is faster in inclined vessel.

Anisotropoc micropolar one-phase fluids: rod-like particles

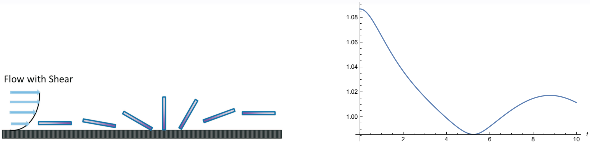


Рис.: (a)Poiselle-like flow $p_x = \text{const}$. (b) Total fluid flux versus time

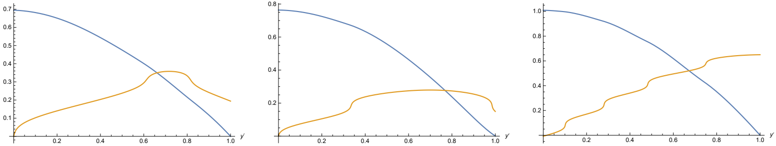


Рис.: Profiles at different instants

$$S_1 = 2\eta_s B_s + 2\eta_a B_a + 2\eta_{an} JB, \quad \frac{d}{dt} J - (\epsilon : \boldsymbol{\omega}) J + J(\epsilon : \boldsymbol{\omega}) = 0,$$

$$B = \nabla \mathbf{v} - \epsilon : \boldsymbol{\omega}$$

Two-phase granular fluids: viscoplasticity effect

$$B_0 = (2\eta_s B_s + 2\eta_a B_a)/(2\eta_s),$$

$$A_0 = (2\beta_s A_s + 2\beta_c \mathrm{tr}\, A \cdot I + 2\beta_a A_a)/(2\beta_s).$$

$$V = \eta_s |\mathrm{B}_0|^2 + \tau_* |\mathrm{B}_0|, \quad V_n = \beta_s |A_0|^2 + \tau_n |A_0|. \tag{1}$$

$$\mathrm{S} \in \partial V(\mathrm{B}_0), \quad \mathrm{N} \in \partial V_n(\mathrm{A}_0). \tag{2}$$

$$\mathrm{S} = \begin{cases} 2\eta_s \mathrm{B}_0 + \tau_* \frac{\mathrm{B}_0}{|\mathrm{B}_0|}, & \mathrm{B}_0(\mathbf{x}, t) \neq 0, \\ \mathrm{S}_p(\mathbf{x}, t), \ |S_p| \leq \tau_*, & \mathrm{B}_0(\mathbf{x}, t) = 0, \end{cases} \tag{3}$$

$$\mathrm{N} = \begin{cases} 2\beta_s A_0 + \tau_n \frac{A_0}{|A_0|}, & A_0(\mathbf{x}, t) \neq 0, \\ \mathrm{N}_p(\mathbf{x}, t), \ |N_p| \leq \tau_*, & A_0(\mathbf{x}, t) = 0. \end{cases} \tag{4}$$

Poiselle-like steady flows in a Hele-Show cell

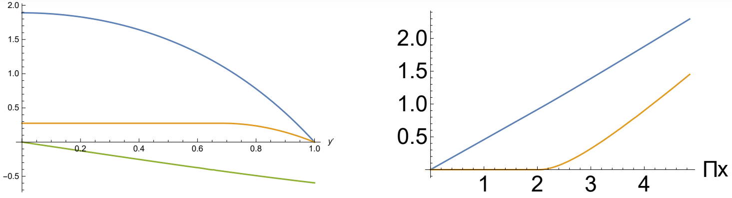


Рис.: (a) Velocity profiles. (b) Total fluid fluxes versus pressure gradient

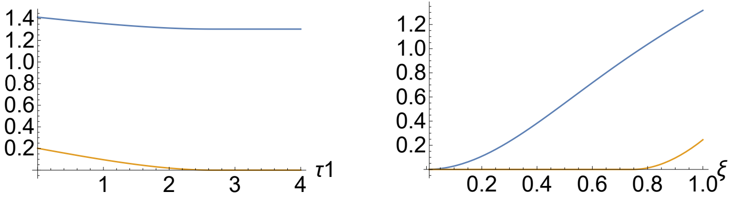


Рис.: (c) Total fluid fluxes versus yield stress. (d) Total fluid fluxes versus channel thickness.

Conclusions

- A mathematical model for two-phase granular fluids is developed and it satisfies the basic principles of **thermodynamics**.
- The model is checked by experimental **benchmarks**: Serge-Silberberg's effect and the Boycott effect.
- Non-spherical particles can inspire the fluid **anisotropy**.
- The model predicts the **bridging effect** due to:

1. fracture's local *bottlenecking*,
2. local *pressure drop* in the fracture (because of leakage),
3. local increase of particle *concentration* in the fracture (since yield stress depends on concentration)