(n,ρ,m) METRIC SPACES

For positive integers n,m with n>m and a subset ρ of the n^{th} symmetric power $M^{(n)}$ of a set M, such that $\Delta_n=\{\underbrace{(x,\dots,x)}_{n} \mid x\in M\}\subseteq\rho$, a map d from $M^{(n)}$ into the set $[0,\infty)$ of nonnegative real numbers is said to be an (n,ρ,m) metric on M, if for each $\mathbf{x}\in M^{(n)}$:

(1) $d(\mathbf{x}) = 0$ if and only if $\mathbf{x}\in\rho$; and

(2) for each $\mathbf{u}\in M^{(m)}$, $d(\mathbf{x}) \leq \Sigma\, d(\mathbf{yu})$, where the sum is over all $\mathbf{y}\in M^{(n-m)}$ such that $\mathbf{x} = \mathbf{yv}$ for some $\mathbf{v}\in M^{(m)}$.

With this notion, a $(2,\Delta_2,1)$ metric is the usual notion of a metric, a $(2,\rho,1)$ metric is the notion of a pseudometric, and the notion of $(n,\rho,1)$ metric is the notion of $(n+1)$ metric defined by K. Menger in the paper Untersuchungen über allgemeine Metrik, Math. Ann. 100, (1928), pp. 75-163.

We investigate the properties of (n,ρ,m) metric spaces M, i.e. the sets equipped with an (n,ρ,m) metric d, with the aim to use them for recognizing images.