Karamata class solutions of Friedman equation

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Abstract

We discuss asymptotic behaviors at infinity of solutions of the system of the following three differential equations:

The Friedman acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right),$$

the fluid equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0,$$

and the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

These equations appear in the study of the expansion scale factor $a(t)$ of the Universe. We discuss this system by use of the theory of regularly varying functions, also known as Karamata functions. As a result we obtain that the solutions of the system might have a multiplicative term which is a slowly varying function. Under usual assumptions for the scale factor $a(t)$, it appears that this slowly varying term exists.